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PHYSICS for you

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Corporate Office:
Plot 99, Sector 44 Institutional area,
Gurgaon -122 003 (HR), Tel : 0124-4951200

Regd. Office
406, Taj Apartment, Near Safdarjung Hospital,
Ring Road, New Delhi - 110029.
e-mail : info@mtg.in website : www.mtg.in

Managing Editor : Mahabir Singh
Editor : Anil Ahlawat (BE, MBA)

Contents

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Editor : Anil Ahlawat

editorial

India is Proud of ISRO

We salute the scientist for their will to attain the goal, undeterred by set-backs and using them as stepping stones for success, the sense of urgency to beat other nations. The great qualities of the leaders of ISRO are responsible for the success of ISRO. They could inspire every scientist to work as a team. These qualities are capped by the faith in God.

Due to geopolitical politics, our friendly nations were imposing embargoe for the know-how and material. However, inspite of the embargo and pressure from U.S.A, USSR supplied about seven cryogenic engines in total. When there were repeated problems, India decided to make our own GSLV engines. This is the one that boosted the morale of India by its success.

PSLV, the earlier model was used to launch the satellites of many other countries which found that Indian launch pads were reliable and affordable. Among the countries who wanted India to launch their vehicle are Japan and U.K. also. Students in India have a wide option for doing research in the field of their choice. We advise the students to learn as much as possible and never leave their research. In the institutions of our country, we can also become leaders.

What is the secret of success of the Chief of ISRO, Dr. Radhakrishnan, Dr. Sreedharan, and the Ex-Chief of our metro and now after retirement, again starting fresh work in many places? It is the faith in God that gave them courage and a will to face any problem. Dr. Radhakrishnan is also a great musician who used to sing in concerts in the temple of Guruvayoor! Many scientists are also great musicians and have strong faith in God.

Let us emulate them, we also salute the leaders who had started these programmes.

Anil Ahlawat
Editor

PHYSICS MUSING

Physics Musing was started in August 2013 issue of Physics For You with the suggestion of Shri Mahabir Singh. The aim of Physics Musing is to augment the chances of bright students preparing for JEE (Main and Advanced) / AIIMS / Other PMTs with additional study material.

In every issue of Physics For You, 10 challenging problems are proposed in various topics of JEE (Main and Advanced) / various PMTs. The detailed solutions of these problems will be published in next issue of Physics For You.

The readers who have solved five or more problems may send their solutions. The names of those who send atleast five correct solutions will be published in the next issue.

We hope that our readers will enrich their problem solving skills through "Physics Musing" and stand in better stead while facing the competitive exams.

By : Akhil Tewari

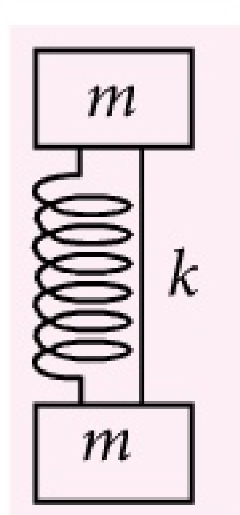
PROBLEM Set 13

SUBJECTIVE PROBLEMS

1 An object of mass 5 kg falls from rest through a vertical distance of 20 m and attains a velocity of 10 m s^{-1}. How much work is done by the resistance of the air on the object? (Take $g = 10$ m s^{-2}).

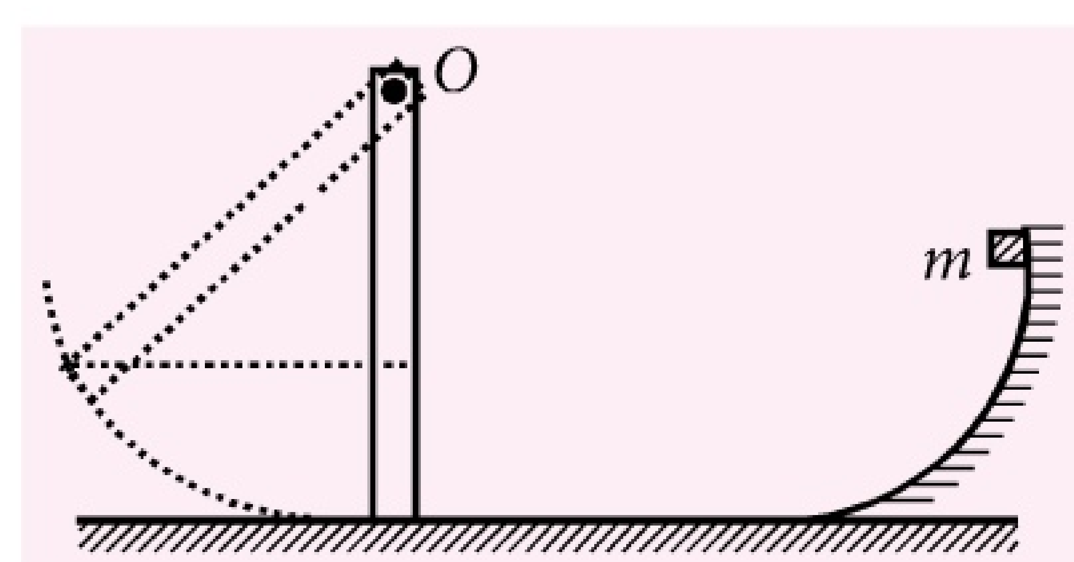
2 A particle of mass m moves along a straight line on smooth horizontal plane, acted upon by a force delivering a constant power P. If the initial velocity of the particle is zero, then find its displacement as a function of time t.

3. A system consists of two identical cubes each of mass m linked together by a compressed massless spring of stiffness k. The cubes are connected by a thread which is burnt at a certain moment. At what value of initial compression ε of the spring will the lower cube bounce up after the thread is burnt.



4. A uniform rod of mass M and length a lies on a smooth horizontal plane. A particle of mass m moving at a speed v perpendicular to the length of the rod strikes it at a distance $a/4$ from the centre and stops after the collision. Find
(a) the velocity of the centre of the rod and
(b) the angular velocity of the rod about its centre just after the collision.

5. In the shown figure a mass m slides down the frictionless surface from height h and collides with the uniform vertical rod of length L and mass M. After collision the mass m sticks to the rod. The rod is free to rotate in a vertical plane about fixed axis through O. Find the maximum angular deflection of the rod from its initial position.



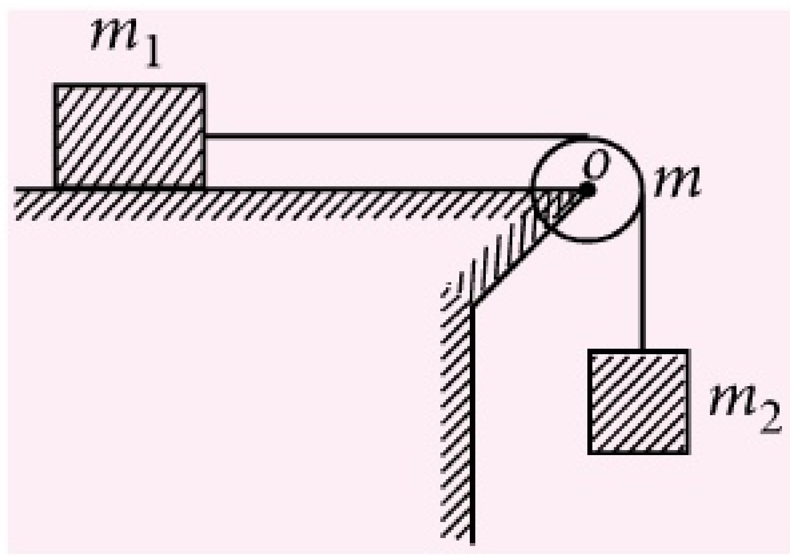
6. In the system shown in the figure masses of the blocks are m_1 and m_2 and that of the pulley (a uniform disc free to rotate about the axle) is m. The co-efficient of friction between the block of mass m_1 and the plane is μ. At $t = 0$ block m_2 starts descending. The string does not slip

Solution Senders of Physics Musing

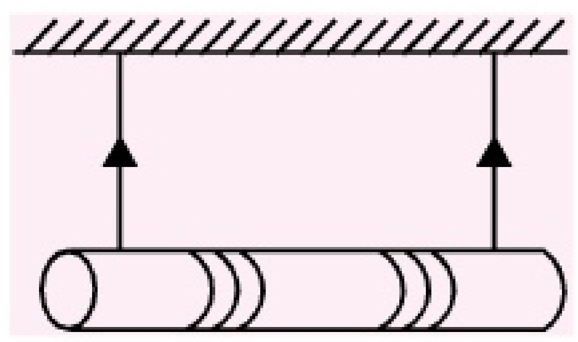
SET-12

1. Arun Nayan, Ghazipur (U.P)
2. Khwaja Sami Boig, Azamgarh (U.P)
3. Saureesh Rao, Nasik (Maharashtra)
4. Shamali Khanna (New Delhi)

on the pulley. Find the work done by the friction acting on the block m_1 over the first t seconds. Neglect the mass of the string and friction in the axle of the pulley.



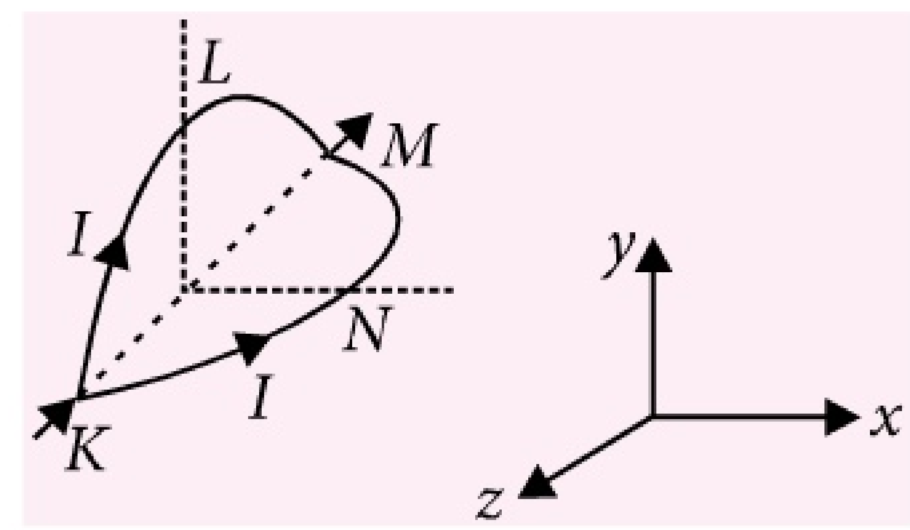
7. A cylinder of mass m is suspended through two strings wrapped around it as shown in figure.



If cylinder is released from rest then find
(a) the tension T in the string and
(b) the speed of the cylinder as it falls through a distance h.

8. A lens has a power of +5 diopter in air. What will be its power if completely immersed in water ? (Given $\mu_g = \frac{3}{2}, \mu_w = \frac{4}{3}$)

9. A circular loop of radius R is bent along a diameter and given a shape as shown in figure. One of the semi-circles (KNM) lies in the x-z plane and the other one (KLM) in the y-z plane with their centres at origin. Current I is flowing through each of the semi-circles as shown in figure.



A particle of charge q is released at the origin with a velocity $\vec{V} = -V_0\,\hat{i}$. Find the instantaneous force $\vec{F}$ on the particle. Assume that space is gravity free.

10. A particle is revolving with a constant angular acceleration α in a circular path of radius r. Find the time when the centripetal acceleration will be numerically equal to the tangential acceleration.

PRACTICE QUESTIONS

TARGET PMTs

Useful for All National and State Level PMTs

1. What will be the unit of time in that system in which the unit of length is metre, unit of mass is kg and unit of force is kg wt?
   (a) $(9.8)^2$ s (b) 9.8 s
   (c) $\sqrt{9.8}$ s (d) $\frac{1}{\sqrt{9.8}}$ s

2. Water drops fall from a tap on the floor 5 m below at regular intervals of time, the first drop striking the floor when the fifth drop begins to fall. The height at which the third drop will be, from ground, at the instant when first drop strikes the ground, will be (Take $g = 10$ m s^{-2})
   (a) 1.25 m (b) 2.15 m
   (c) 2.73 m (d) 3.75 m

3. A block of mass 5 kg resting on a horizontal surface is connected by a cord passing over a light frictionless pulley to a hanging block of mass 5 kg. The coefficient of kinetic friction between the block and the surface is 0.5. Tension in the cord is (Take $g = 9.8$ m s^{-2})
   (a) 49 N (b) 36 N
   (c) 36.75 N (d) 2.45 N

4. A 100 eV electron is fired directly towards a large metal plate having surface charge density -2×10^{-6} C m^{-2}. The distance from where the electron be projected so that it just fails to strike the plate is
   (a) 0.22 mm (b) 0.44 mm
   (c) 0.66 mm (d) 0.88 mm

5. An electric cable of copper has just one wire of radius 9 mm. Its resistance is 5 Ω. This single copper wire of cable is replaced by 6 different well insulated copper wires each of radius 3 mm. The total resistance of the cable will now be equal to
   (a) 7.5 Ω (b) 45 Ω
   (c) 90 Ω (d) 270 Ω

6. The torque required to hold a small circular coil of 10 turns, 2×10^{-4} m^2 area and carrying 0.5 A current in the middle of a long solenoid of 10^3 turns m^{-1} carrying 3 A current, with its axis perpendicular to the axis of the solenoid, is
   (a) $12\pi \times 10^{-7}$ N m (b) $6\pi \times 10^{-7}$ N m
   (c) $4\pi \times 10^{-7}$ N m (d) $2\pi \times 10^{-7}$ N m

7. A body of mass m slides down an inclined plane and reaches the bottom with a velocity v. If the same mass were in the form of a ring which rolls down this incline, the velocity of the ring at the bottom would be
   (a) v (b) $\frac{v}{\sqrt{2}}$
   (c) $\sqrt{2}v$ (d) $\sqrt{\frac{2}{5}}\,v$

8. A bullet when fired at a target with a speed of 100 m s^{-1}, penetrates 1 m into it. If the bullet is fired at a similar target with a thickness of 25 cm, then it will emerge with a velocity (in m s^{-1}) of
   (a) $100 \times \frac{1}{\sqrt{2}}$ (b) $100 \times \frac{2}{\sqrt{3}}$
   (c) $100 \times \frac{\sqrt{3}}{2}$ (d) $100 \times \frac{2}{\sqrt{5}}$

9. A metal sphere of radius r and specific heat s is rotated about an axis passing through its centre at a speed of υ rotations per second. It is suddenly stopped and 50% of its energy is used in increasing its temperature. Then the rise in temperature of the sphere is

(a) $\frac{2\pi^2\upsilon^2 r^2}{5s}$ (b) $\frac{\pi^2\upsilon^2}{10r^2 s}$

(c) $\frac{7}{8}\pi r^2\upsilon^2 s$ (d) $\frac{5(\pi r\upsilon)^2}{14s}$

10. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4.0×10^{-2} kg m^{-1}. What is the speed of a transverse wave on the string?

(a) 69 m s^{-1} (b) 79 m s^{-1}

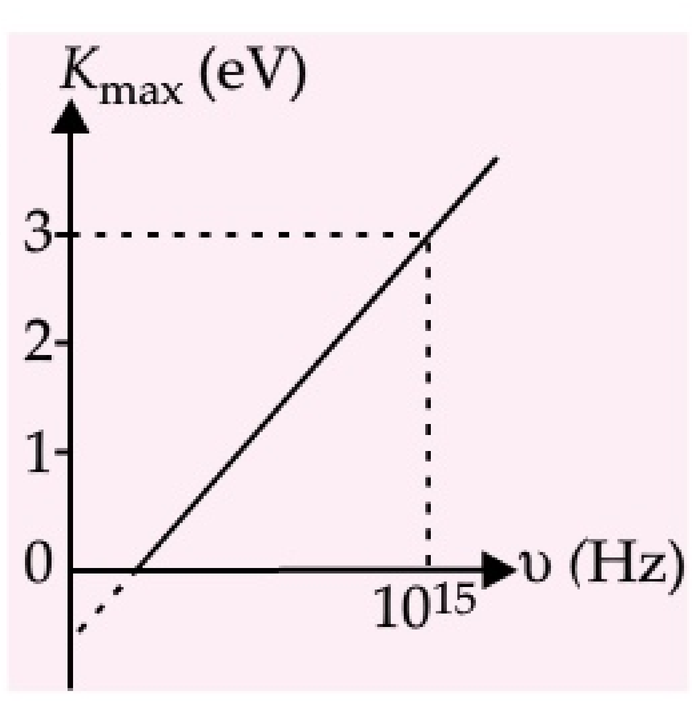
(c) 89 m s^{-1} (d) 99 m s^{-1}

11. In a Young's double slit experiment, $d = 0.5$ mm and $D = 100$ cm. It is found that 9th bright fringe is at a distance of 7.5 mm from the second dark fringe of fringe pattern. The wavelength of light used is (in Å)

(a) $\frac{2500}{7}$ (b) 2500

(c) 5000 (d) $\frac{5000}{7}$

12. Figure represents a graph of kinetic energy of most energetic photoelectrons K_{max} (in eV) and frequency υ for a metal used as cathode in photoelectric experiment. The threshold frequency of light for the photoelectric emission from the metal is



(a) 1×10^{14} Hz

(b) 1.5×10^{14} Hz

(c) 2.1×10^{14} Hz

(d) 2.8×10^{14} Hz

13. A man throws a ball at an angle of 45° with the horizontal plane from a height of 15 m. If the shot strikes the ground at a horizontal distance of 30 m, the velocity of throw is (Take $g = 10$ m s^{-2})

(a) 10 m s^{-1} (b) $10\sqrt{2}$ m s^{-1}

(c) 20 m s^{-1} (d) $20\sqrt{2}$ m s^{-1}

14. Force on a 1 kg mass on earth of radius R is 10 N. Then the force on a satellite revolving around the earth in the mean orbital radius $3R/2$ will be (Take mass of satellite = 100 kg)

(a) 4.44×10^2 N (b) 3.33×10^2 N

(c) 500 N (d) 6.66×10^2 N

15. An electron tube was sealed off during manufacture at a pressure of 1.2×10^{-7} mm of mercury at 27°C. Its volume is 100 cm^3. The number of molecules that remain in the tube is (Take $g = 10$ m s^{-2})

(a) 2×10^{16} (b) 3×10^{15}

(c) 4×10^{11} (d) 5×10^{11}

16. Three designs are proposed for an engine operating between 500 K and 300 K. For 1 kcal of heat input, design A claims to produce 3000 J of work, design B claims to produce 2000 J of work and design C claims to produce 1000 J of work. The design which is possible is

(a) A only (b) B only

(c) All (d) C only

17. A body weigh 50 g in air and 40 g in water. How much would it weigh in a liquid of specific gravity 1.5?

(a) 65 g (b) 45 g

(c) 30 g (d) 35 g

18. The work done by electric field during the displacement of a negatively charged particle towards a fixed positively charged particle is 9 J. As a result the distance between the charges has been decreased by half. What work is done by the electric field over the first half of this distance?

(a) 3 J (b) 6 J

(c) 1.5 J (d) 9 J

19. Two co-axial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area A = 10 cm^2 and length = 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is

(a) $2.4\pi \times 10^{-4}$ H (b) $2.4\pi \times 10^{-5}$ H

(c) $4.8\pi \times 10^{-4}$ H (d) $4.8\pi \times 10^{-5}$ H

20. Two circular coils are made of two identical wires of same length and carry same current. If the number of turns of two coils are 4 and 2, then the ratio of magnetic induction at the centres will be

(a) 4 : 1 (b) 2 : 1
(c) 1 : 2 (d) 1 : 1

21. A thin lens (μ = 1.5) of focal length + 10 cm is immersed in water (μ = 1.33). The new focal length is

(a) 20 cm (b) 40 cm
(c) 48 cm (d) 12 cm

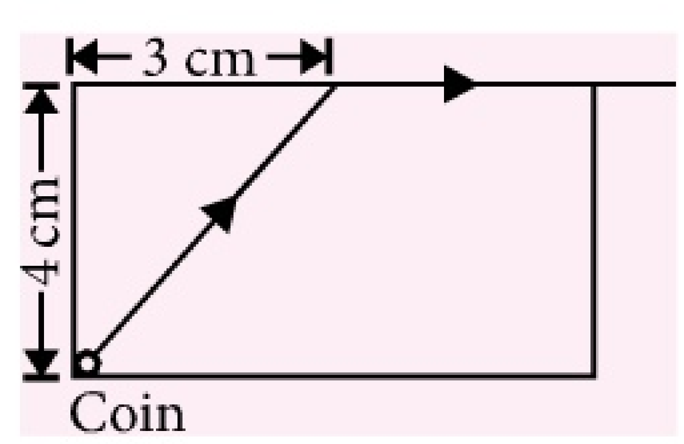
22. The counting rate observed from a radioactive source at t = 0 was 1600 counts s^{-1}, and at t = 8 s, it was 100 counts s^{-1}. The counting rate observed as counts s^{-1} at t = 6 s will be

(a) 400 (b) 300
(c) 250 (d) 200

23. In a common emitter transistor, the base current I_B = 2 μA, α = 0.9, then I_B is equal to

(a) 18 μ A (b) 20 μ A
(c) 22 μ A (d) 24 μ A

24. A small coin is resting on the bottom of a beaker filled with liquid. A ray of light from the coin travels upto the surface of the liquid and moves along its surface. How fast is the light travelling in the liquid?



(a) 2.4×10^8 m s^{-1}
(b) 3.0×10^8 m s^{-1}
(c) 1.2×10^8 m s^{-1}
(d) 1.8×10^8 m s^{-1}

25. A thin prism P_1 with 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is

(a) 5.33° (b) 4°
(c) 3° (d) 2.6°

26. An energy of 24.6 eV is required to remove one of the electrons from a neutral helium atom. The energy (in eV) required to remove both the electrons from a neutral helium atom is

(a) 38.2 (b) 49.2
(c) 51.8 (d) 79.0

27. The current gain of a transistor in common-emitter mode is 49. The change in collector current and emitter current corresponding to the change in base current by 5.0 μA will be

(a) 245 μA, 250 μA (b) 240 μA, 235 μA
(c) 260 μA, 255 μA (d) none of these

28. A 15 g ball is shot from a spring gun whose spring has a force constant of 600 N m^{-1}. The spring is compressed by 5 cm. The greatest possible horizontal range of the ball for this compression is (Take g = 10 m s^{-2})

(a) 6.0 m (b) 10.0 m
(c) 12.0 m (d) 8.0 m

29. The condition of apparent weightlessness can be created momentarily when a plane flies over the top of a vertical circle. At a speed of 900 km h^{-1}, the radius of the vertical circle that pilot must use is

(a) 10.6 km (b) 8.5 km
(c) 6.4 km (d) 4.0 km

30. A uniform disc of mass m and radius r is rolling down a rough inclined plane of inclination 30°. The coefficient of static and dynamic friction each = μ and the only forces acting are gravitational and frictional. The magnitude of frictional force acting on the disc is

(a) mg (b) $\frac{mg}{2}$
(c) $\frac{mg}{3}$ (d) $\frac{mg}{6}$

31. The ratio of the energy required to raise a satellite upto a height h above the earth of radius R to that the kinetic energy of the satellite into that orbit is

(a) $R : h$ (b) $h : R$
(c) $R : 2h$ (d) $2h : R$

32. A satellite is moving in a circular orbit at a certain height above the earth's surface. It takes 5.26×10^3 s to complete one revolution with a centripetal acceleration equal to 9.32 m s^{-2}. The height of the satellite orbit above the earth's surface is (Take radius of earth = 6.37×10^6 m)

(a) 70 km (b) 170 km
(c) 190 km (d) 220 km

33. Displacement-time equation of a particle executing SHM is, $x = 4 \sin \omega t + 3 \sin (\omega t + \pi/3)$. Here x is in centimeters and t in seconds. The amplitude of oscillation of the particle is approximately

(a) 5 cm (b) 6 cm
(c) 7 cm (d) 9 cm

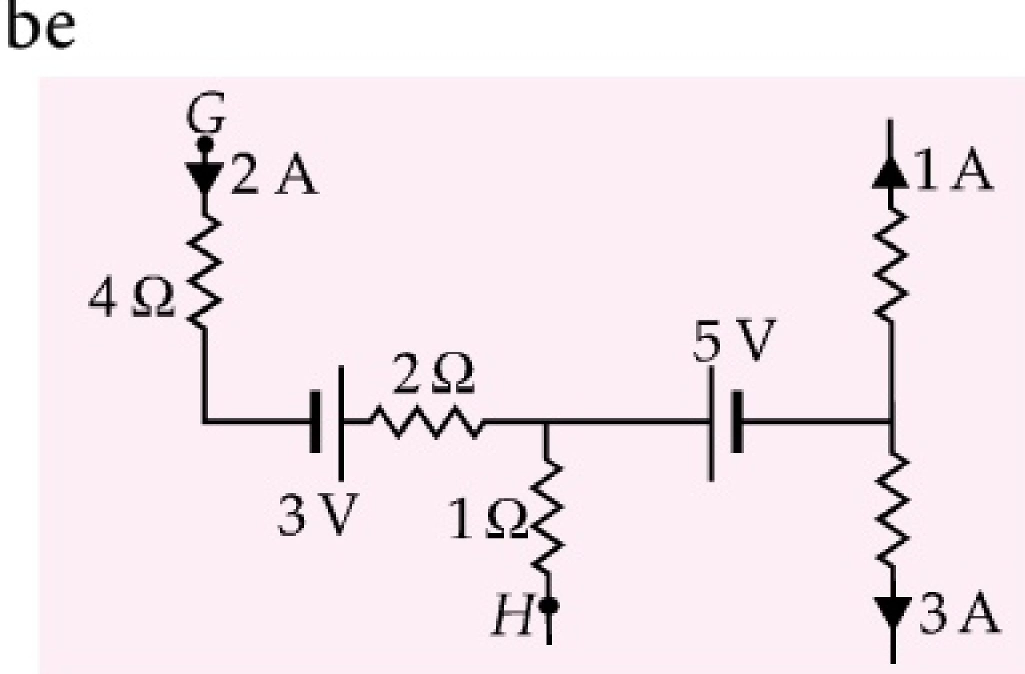
34. A gas under constant pressure of 4.5×10^5 Pa when subjected to 800 kJ of heat, changes the volume from 0.5 m^3 to 2.0 m^3. The change in internal energy of the gas is

(a) 6.75×10^5 J (b) 5.25×10^5 J
(c) 3.25×10^5 J (d) 1.25×10^5 J

35. Two tuning forks A and B vibrating simultaneously produce 5 beats. Frequency of B is 512 Hz. It is seen that if one arm of A is filed, then the number of beats increases. Frequency of A will be

(a) 502 Hz (b) 507 Hz
(c) 517 Hz (d) 522 Hz

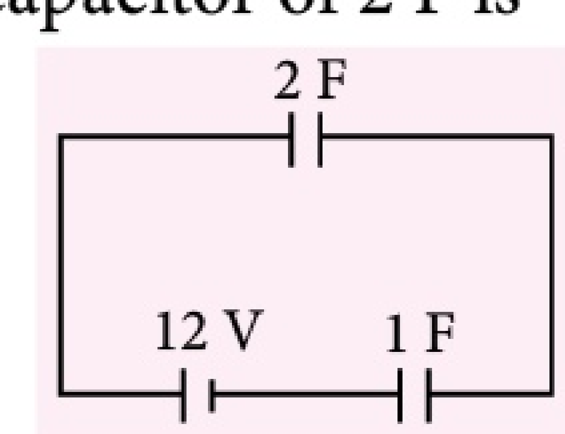
36. In the part of a circuit shown in figure, the potential difference between points G and H will be



(a) 0 V (b) 12 V
(c) 7 V (d) 3 V

37. In a circuit shown in figure, the potential difference across the capacitor of 2 F is

(a) 8 V
(b) 4 V
(c) 12 V
(d) 6 V



38. A parrot sitting on the floor of a wire cage which is being carried by a boy, starts flying. The boy will feel that the cage is now

(a) heavier (b) lighter
(c) shows no change in weight
(d) lighter in the beginning and heavier later.

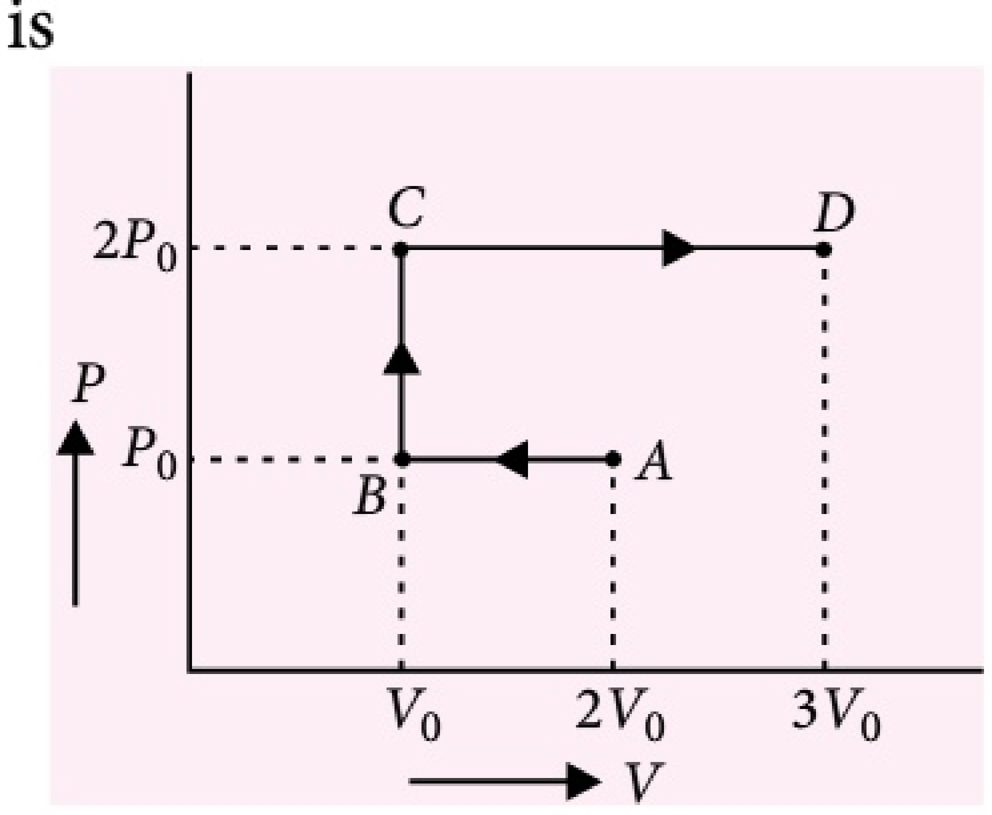
39. A sinusoidal voltage of peak value 283 V and frequency 50 Hz is applied to a series LCR circuit in which R = 3 Ω, L = 25.48 mH, and C = 796 μF. The impedance of the circuit is

(a) 4 Ω (b) 5 Ω
(c) 6 Ω (d) 7 Ω

40. The rms value of the electric field of the light coming from the sun is 720 N C^{-1}. The average total energy density of the electromagnetic wave is

(a) 3.33×10^{-3} J m^{-3} (b) 4.58×10^{-6} J m^{-3}
(c) 6.37×10^{-9} J m^{-3} (d) 81.35×10^{-12} J m^{-3}.

41. P-V diagram of an ideal gas is as shown in figure. Work done by the gas in the process $ABCD$ is



(a) $4P_0 V_0$ (b) $2P_0 V_0$
(c) $3P_0 V_0$ (d) $P_0 V_0$

42. The resistance of a bulb filament is 100 Ω at a temperature of 100°C. If its temperature coefficient of resistance be 0.005 per °C, its resistance will become 200 Ω at a temperature of

(a) 300°C (b) 400°C
(c) 500°C (d) 200°C

43. Two long parallel wires carry equal current I flowing in the same direction are at a distance $2d$ apart. The magnetic field B at a point lying on the perpendicular line joining the wires and at a distance x from the midpoint is

(a) $\dfrac{\mu_0 Id}{\pi(d^2+x^2)}$ (b) $\dfrac{\mu_0 Ix}{\pi(d^2-x^2)}$

(c) $\dfrac{\mu_0 Ix}{(d^2+x^2)}$ (d) $\dfrac{\mu_0 Id}{(d^2-x^2)}$

44. A block of wood is kept on the floor of a stationary elevator. The elevator is beginning to descend with an acceleration of 12 m s^{-2}. If

$g = 10$ m s^{-2}, then the displacement of the block during first 0.2 s after start is

(a) 0.02 m (b) 0.2 m
(c) 0.1 m (d) 0.04 m

45. A simple pendulum 1 m long has a bob of 10 kg. If the pendulum swings from a horizontal position, the K.E. of the bob, at the instant it passes through the lowest position of its path is

(a) 89 J (b) 95 J
(c) 98 J (d) 85 J

SOLUTIONS

1. (d): As $[F] = [MLT^{-2}]$

$$\therefore \quad T^2 = \frac{ML}{F} = \frac{1\text{ kg}\times 1\text{ m}}{1\text{ kg wt}} = \frac{1\text{ kg}\times 1\text{ m}}{9.8\text{ N}}$$

$(\because\ 1 \text{ kg wt} = 9.8 \text{ N})$

or $\quad T = \dfrac{1}{\sqrt{9.8}}$ s

2. (d): By the time 5$^{\text{th}}$ water drop starts falling, the first water drop reaches the ground.

As $\quad u = 0,\ h = \dfrac{1}{2}gt^2 = \dfrac{1}{2}\times 10\times t^2$

or $\quad 5 = \dfrac{1}{2}\times 10\times t^2$ or $t = 1$ s

Hence, the interval of each water drop

$$= \frac{1\text{ s}}{4} = 0.25 \text{ s}$$

When the 5$^{\text{th}}$ drop starts its journey towards ground, the third drop travels in air for

$t_1 = 0.25 + 0.25 = 0.5$ s

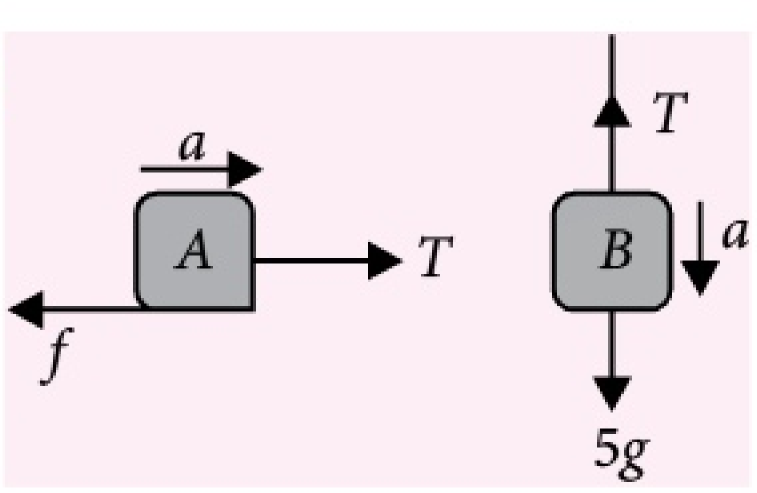
$\therefore$ Distance covered by 3$^{\text{rd}}$ drop in air is

$$h_1 = \frac{1}{2}gt_1^2 = \frac{1}{2}\times 10\times (0.5)^2$$
$$= 5 \times 0.25 = 1.25 \text{ m}$$

So, third water drop will be at a height of

$= 5 - 1.25 = 3.75$ m

3. (c):



From free body diagram of block B

$5g - T = 5a$ or $T = 5g - 5a$ ...(i)

From free body diagram of block A

$T - f = 5a$ or $T - 5\mu g = 5a$

or $\quad 5g - 5a - 5 \times 0.5 \times g = 5a$ (using (i))

or $\quad 10a = 2.5g = 2.5 \times 9.8$

or $\quad a = 0.25 \times 9.8 = 2.45$ m s^{-2}

$\therefore \quad T = (5 \times 9.8 - 5 \times 2.45)$ N (from (i))

$= (49 - 12.25)$ N $= 36.75$ N

4. (b): Here, K.E. = 100 eV = $100 \times 1.6 \times 10^{-19}$ J. This is lost when electron moves through a distance d towards the negative plate.

$$\therefore \text{K.E.} = \text{work done} = F\times s = qE\times s = e\left(\frac{\sigma}{\varepsilon_0}\right)d$$

or $\quad d = \dfrac{(\text{K.E.})\varepsilon_0}{e\sigma}$

$$d = \frac{100\times 1.6\times 10^{-19}\times 8.85\times 10^{-12}}{1.6\times 10^{-19}\times 2\times 10^{-6}}$$
$$= 4.425 \times 10^{-4} \text{ m} = 0.4425 \text{ mm}$$

5. (a): For one wire cable,

Resistance, $R = \dfrac{\rho l}{A} = \dfrac{\rho l}{\pi(9\times 10^{-3})^2} = 5$

or $\quad \rho l = 5\pi(9 \times 10^{-3})^2$

For other wire of cable,

$$\text{Resistance, } R' = \frac{\rho l}{\pi(3\times 10^{-3})^{-2}}$$
$$= \frac{5\pi(9\times 10^{-3})^2}{\pi(3\times 10^{-3})^2} = 45\ \Omega$$

When six wires each of resistance R' are connected in parallel, their effective resistance will be

$$R_p = \frac{R'}{6} = \frac{45}{6} = 7.5\ \Omega$$

6. (a): Magnetic dipole moment of current loop is

$M = NIA = 10 \times 0.5 \times 2 \times 10^{-4} = 10^{-3}$ A m^2

Magnetic field inside the solenoid carrying current is

$B = \mu_0 nI = 4\pi \times 10^{-7} \times 10^3 \times 3$
$= 12\pi \times 10^{-4}$ T

$\therefore \quad$ Torque, $\tau = MB \sin\theta$
$= 10^{-3} \times 12\pi \times 10^{-4} \times \sin 90°$
$(\because\ \theta = 90°)$
$= 12\pi \times 10^{-7}$ N m

7. (b): Kinetic energy of translation of mass m sliding down the incline with velocity $v = \dfrac{1}{2}mv^2$.

Total kinetic energy of the ring rolling down the incline with velocity v_{cm}

$$=\frac{1}{2}mv_{cm}^2+\frac{1}{2}I\omega^2$$

$$=\frac{1}{2}mv_{cm}^2+\frac{1}{2}(mr^2)\left(\frac{v_{cm}}{r}\right)^2=mv_{cm}^2$$

Thus, $mv_{cm}^2=\frac{1}{2}mv^2$ or $v_{cm}=\frac{v}{\sqrt{2}}$

8. (c) : In the first case, as $v_f = 0$,

$\therefore$ According to work-energy theorem

$$0-\frac{1}{2}mv_i^2=-F\times 1 \text{ or } \frac{1}{2}mv_i^2=F$$

In the second case,

$$\frac{1}{2}mv_f^2=\frac{1}{2}mv_i^2-\left(F\times\frac{1}{4}\right)$$

$$=F-\frac{F}{4}=\frac{3}{4}F=\frac{3}{4}\left(\frac{1}{2}mv_i^2\right)$$

or $\quad v_f=\frac{\sqrt{3}}{2}v_i=100\left(\frac{\sqrt{3}}{2}\right)\text{ m s}^{-1}$

9. (a) : Moment of inertia of a sphere,

$$I=\frac{2}{5}mr^2$$

$\omega = 2\pi\upsilon$ rad s^{-1}

$\therefore$ Kinetic energy $=\frac{1}{2}I\omega^2$

$$=\frac{1}{2}\times\frac{2}{5}mr^2\times(2\pi\upsilon)^2=\frac{4}{5}m\pi^2r^2\upsilon^2$$

Half of this energy is converted into heat.

$$\therefore\quad \Delta Q=\frac{1}{2}\times\frac{4}{5}m\pi^2r^2\upsilon^2=\frac{2}{5}m\pi^2r^2\upsilon^2 \quad ...(i)$$

Specific heat, $s=\frac{1}{m}\frac{\Delta Q}{\Delta T}$

or $\quad \Delta T=\frac{\Delta Q}{ms}=\frac{\frac{2}{5}m\pi^2r^2\upsilon^2}{ms}$ (Using (i))

$$\Delta T=\frac{2\pi^2r^2\upsilon^2}{5s}$$

10. (b) : Here, $\upsilon = 45$ Hz, $M = 3.5\times10^{-2}$ kg

Linear mass density, $\mu=\frac{\text{mass}}{\text{length}}$

$= 4.0\times10^{-2}$ kg m^{-1}

$$\therefore\quad l=\frac{M}{\mu}=\frac{3.5\times10^{-2}}{4.0\times10^{-2}}=\frac{7}{8}$$

As $\frac{\lambda}{2}=l=\frac{7}{8}\quad\therefore\quad\lambda=\frac{7}{4}$ m

The speed of the transverse wave is

$$v=\upsilon\lambda=45\times\frac{7}{4}\text{ m s}^{-1}\approx 79\text{ m s}^{-1}$$

11. (c) : For bright fringes, $x=\frac{n\lambda D}{d}$

where n = 0, 1, 2, 3, ...

For dark fringes, $x=\frac{(2n-1)\lambda D}{2d}$

where n = 1, 2, 3, ...

As per question

$$\frac{9\lambda D}{d}-\frac{3\lambda D}{2d}=7.5\times10^{-3}\text{ m}$$

or $\quad\frac{15\lambda D}{2d}=7.5\times10^{-3}$ m

or $\quad\lambda=\frac{2\times7.5\times10^{-3}\text{ m}\times d}{15D}$

Substituting the given values, we get

$$\lambda=\frac{2\times7.5\times10^{-3}\text{ m}\times0.5\times10^{-3}\text{ m}}{15\times1\text{ m}}$$

$= 0.5\times10^{-6}$ m = 5000 Å

12. (d) : From graph, for $\upsilon = 10^{15}$ Hz,

K_{max} = 3 eV = $3\times1.6\times10^{-19}$ J

According to Einstein's photoelectric equation

$K_{max} = h\upsilon - h\upsilon_0$

or $\quad h\upsilon_0 = h\upsilon - K_{max}$

or $\quad\upsilon_0=\upsilon-\frac{K_{max}}{h}=10^{15}-\frac{3\times1.6\times10^{-19}}{6.63\times10^{-34}}$

$= (10 - 7.2)\times10^{14}$

$= 2.8\times10^{14}$ Hz

13. (b) : For horizontal motion, $x = u\cos\theta t$

so, $30 = u\cos 45°t=\frac{u}{\sqrt{2}}t$ ...(i)

For vertical downward motion,

$$y=-u\sin45°t+\frac{1}{2}gt^2$$

or $\quad 15=-\frac{u}{\sqrt{2}}t+\frac{1}{2}\times10\times t^2$

or $\quad 15 = -30 + 5t^2$

or $\quad t = 3$ s (Using (i))

$$\therefore\quad u=\frac{30\sqrt{2}}{t}=\frac{30\sqrt{2}}{3}=10\sqrt{2}\text{ m s}^{-1}$$

14. (a) : On the surface of earth, the force on a mass of 1 kg is

$$F = \frac{GMm}{R^2} = \frac{GM \times 1}{R^2} = 10 \quad \text{...(i)}$$

When the mean orbital radius of the satellite, $r = 3R/2$, the force on the satellite is

$$F' = \frac{GMm'}{r^2} = \frac{GM \times 100}{(3/2)^2 R^2} \quad (\because\ m' = 100 \text{ kg})$$

$$= \frac{10 \times 4 \times 100}{9} = 4.44 \times 10^2 \text{ N} \quad \text{(Using (i))}$$

15. (c) : Here, $P = 1.2 \times 10^{-7}$ mm of Hg

$P = 1.2 \times 10^{-7} \times 10^{-3} \times (13.6 \times 10^3) \times 10 \text{ N m}^{-2}$

$= 1.2 \times 13.6 \times 10^{-6} \text{ N m}^{-2}$

$V = 100 \text{ cm}^3 = 100 \times 10^{-6} \text{ m}^3$

$T = 27°\text{C} = 300 \text{ K}$

From ideal gas equation, number of moles, $n = \frac{PV}{RT}$

$$= \frac{(1.2 \times 13.6 \times 10^{-6}) \times (100 \times 10^{-6})}{8.31 \times 300}$$

$= 0.65 \times 10^{-12}$

$\therefore$ Number of molecules $= nN_A$

$= 0.65 \times 10^{-12} \times 6.023 \times 10^{23}$

$\approx 4 \times 10^{11}$

16. (d) : Maximum value of efficiency

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{500} = \frac{2}{5}$$

As $\eta = \frac{W}{Q_1}$

$\therefore$ $W = \eta Q_1 = \frac{2}{5} \times 1000$ cal

$= 400 \times 4.2 \text{ J} = 1680 \text{ J}$

As no engine can produce more than 1680 J, designs A and B are not possible.

17. (d) : Density of water, $\rho_w = 1 \text{ g cc}^{-1}$

Density of liquid, ρ_l

= specific gravity of liquid × density of water

$= 1.5 \times 1 = 1.5 \text{ g cc}^{-1}$.

If V is the volume of the body, then

Weight of body in water = Weight of body in air – Upthrust of liquid

$40g = 50g - V \times \rho_w \times g$

or $V = 50 - 40 = 10$ cc

Weight of body in liquid = weight of body in air – upthrust of liquid

$= 50g - V\rho_l g = (50 - 10 \times 1.5)g$

$= 35g$

$\therefore$ It will weigh 35 g.

18. (a) : Here, $U_1 = \frac{Q(-q)}{4\pi\varepsilon_0 r}$, $U_2 = \frac{Q(-q)}{4\pi\varepsilon_0 (r/2)}$

$$\therefore \quad U_1 - U_2 = \frac{Q(-q)}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{2}{r}\right]$$

$$= \frac{Qq}{4\pi\varepsilon_0 r} = 9 \quad \text{...(i)}$$

When negative charge travels first half of distance, *i.e.*, $r/4$, potential energy of the system

$$U_3 = \frac{Q(-q)}{4\pi\varepsilon_0 (3r/4)} = \frac{-Qq}{4\pi\varepsilon_0 r} \times \frac{4}{3}$$

$\therefore$ Work done $= U_1 - U_3$

$$= \frac{Q(-q)}{4\pi\varepsilon_0 r} + \frac{Qq}{4\pi\varepsilon_0 r} \times \frac{4}{3}$$

$$= \frac{Qq}{4\pi\varepsilon_0 r} \times \frac{1}{3} = \frac{9}{3} = 3 \text{ J}$$

19. (a) : Here, $A = 10 \text{ cm}^2 = 10 \times 10^{-4} \text{ m}^2 = 10^{-3} \text{ m}^2$

$l = 20 \text{ cm} = 0.2 \text{ m}$, $N_1 = 300$, $N_2 = 400$

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

$$= \frac{(4\pi \times 10^{-7}) \times 300 \times 400 \times 10^{-3}}{0.2}$$

$= 2.4\pi \times 10^{-4}$ H

20. (a) : Here, $l = N_1 2\pi r_1 = N_2 2\pi r_2 \;\therefore\; N_1 r_1 = N_2 r_2$

$$\frac{r_1}{r_2} = \frac{N_2}{N_1}$$

As, $B = \frac{\mu_0 NI}{2r}$ $\therefore$ $\frac{B_1}{B_2} = \frac{\mu_0 N_1 I/2r_1}{\mu_0 N_2 I/2r_2}$

$$= \frac{N_1}{N_2} \cdot \frac{r_2}{r_1} = \left(\frac{N_1}{N_2}\right)^2 = \left(\frac{4}{2}\right)^2 = \frac{4}{1}$$

21. (b) : When lens is in air, then

$$\frac{1}{f} = ({}^a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{10} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

or $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{5}$

When lens is in water, then

$$\frac{1}{f'}=({}^w\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{f'}=\left(\frac{{}^a\mu_g}{{}^a\mu_w}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$=\left(\frac{1.5}{1.33}-1\right)\times\frac{1}{5}=\frac{1}{40}$$

or $f' = 40$ cm

22. (d): As $\frac{N}{N_0}=\left(\frac{1}{2}\right)^n$

$\therefore \left(\frac{1}{2}\right)^n=\frac{100}{1600}=\frac{1}{16}=\left(\frac{1}{2}\right)^4$

or $n = 4$

From $t=nT_{1/2}$, $T_{1/2}=\frac{t}{n}=\frac{8}{4}=2$ s

At $t=6$ s, $n=\frac{6}{2}=3$

$\therefore \frac{N}{N_0}=\left(\frac{1}{2}\right)^3=\frac{1}{8}$

$N=\frac{N_0}{8}=\frac{1600}{8}=200$ counts s^{-1}

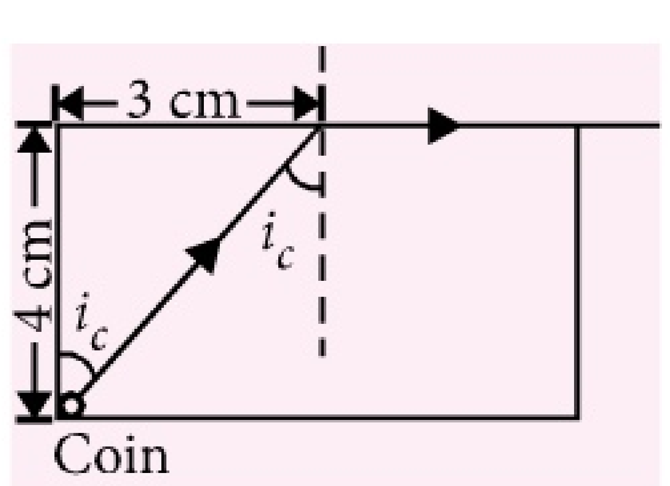
23. (a) : As $\beta=\frac{\alpha}{1-\alpha}=\frac{0.9}{1-0.9}=9$

$I_C = \beta I_B = 9 \times 2\ \mu A = 18\ \mu A$

24. (d): As $\sin i_c=\frac{3}{\sqrt{3^2+4^2}}=\frac{3}{5}$

$\therefore \mu=\frac{1}{\sin i_c}=\frac{5}{3}$

(as i_c is the angle which the ray from the coin makes with 4 cm side)



As $\mu=\frac{c}{v_l}$

$\therefore v_l=\frac{c}{\mu}=\frac{3\times10^8\ \text{m s}^{-1}}{(5/3)}=1.8\times10^8\ \text{m s}^{-1}$

25. (c) : For dispersion without deviation

$\delta_1 + \delta_2 = 0$, *i.e.*, $\delta_1 = -\delta_2$

$\therefore (\mu_1 - 1)A_1 = -(\mu_2 - 1)A_2$

Here, $\mu_1 = 1.54$, $\mu_2 = 1.72$, $A_1 = 4°$

$(1.54 - 1)4° = -(1.72 - 1)A_2$

$\therefore A_2 = -3°$

Hence, angle of prism P_2 is 3°. Negative sign shows that prism P_2 is inverted with respect to prism P_1.

26. (d): For He$^+$ ion, $E_n=\frac{13.6}{n^2}(2)^2$ eV

(after removal of an electron from He atom)

$E_1=-\frac{13.6\times4}{1^2}$ eV $=-54.4$ eV

Thus, energy required to remove the second electron = 54.4 eV

$\therefore$ Total energy required to remove both the electrons

= 24.6 eV + 54.4 eV = 79.0 eV

27. (a) : Here, $\beta = 49$, $\Delta I_B = 5.0\ \mu A$

$\therefore \Delta I_C = \beta\Delta I_B = 49 \times 5.0\ \mu A = 245\ \mu A$

$\Delta I_E = \Delta I_B + \Delta I_C = 5.0\ \mu A + 245\ \mu A = 250\ \mu A$

28. (b): As $R_{max}=\frac{u^2}{g}=\left(\frac{1}{2}mu^2\right)\left(\frac{2}{mg}\right)$

But $\frac{1}{2}mu^2=\frac{1}{2}kx^2$

$\therefore R_{max}=\left(\frac{1}{2}kx^2\right)\left(\frac{2}{mg}\right)=\frac{kx^2}{mg}$

$$=\frac{600(5\times10^{-2})^2}{15\times10^{-3}\times10}=10.0\ \text{m}$$

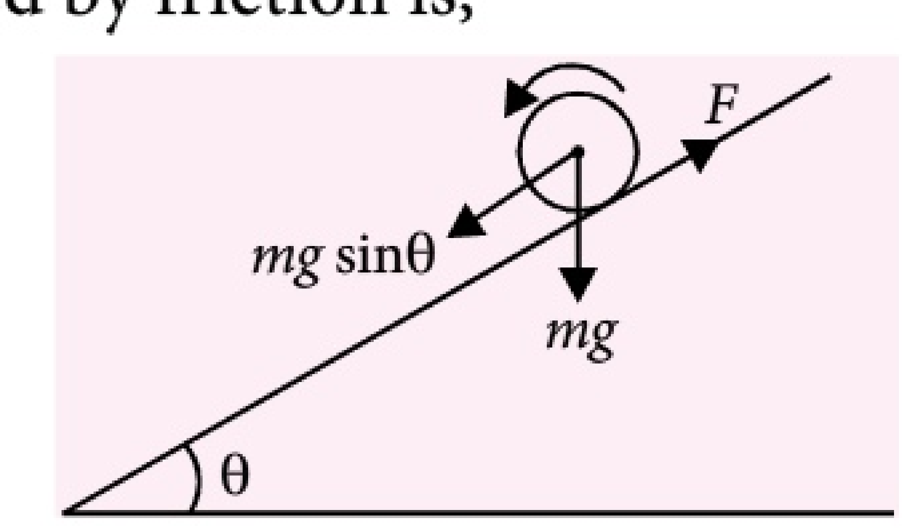
29. (c) : Here, $v = 900$ km h^{-1} $=\frac{900\times1000}{60\times60}$ $= 250$ m s^{-1};

$g = 9.8$ m s^{-2}

For apparent weightlessness, $\frac{mv^2}{r}=mg$

or $r=\frac{v^2}{g}=\frac{250\times250}{9.8}=6377.5\ \text{m}\approx6.4\ \text{km}$

30. (d): If F is the force of friction, then torque produced by friction is,



$$F \times r = I\alpha = \frac{1}{2}mr^2\alpha = \frac{1}{2}mr(r\alpha) = \frac{1}{2}mra$$

$$\left(\because I = \frac{1}{2}mr^2 \text{ and } a = r\alpha\right)$$

$$\therefore \quad F = \frac{1}{2}ma$$

But $a = \dfrac{g\sin\theta}{1+K^2/R^2} = \dfrac{g\sin 30°}{1+1/2} = \dfrac{g}{3}$

$$\therefore \quad F = \frac{1}{2}ma = \frac{1}{2}m\frac{g}{3} = \frac{mg}{6}$$

31. (d): Energy required to raise the satellite to a height h from surface of earth

$$E_1 = -\frac{GMm}{(R+h)} - \left(-\frac{GMm}{R}\right) = \frac{GMmh}{R(R+h)}$$

Kinetic energy of satellite,

$$E_2 = \frac{1}{2}mv_o^2 = \frac{1}{2}m\frac{GM}{(R+h)} \qquad \left(\because v_o = \sqrt{\frac{Gm}{(R+h)}}\right)$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{2h}{R}$$

32. (b): As, $T = 2\pi\sqrt{\dfrac{(R+h)^3}{MG}}$

or $\quad \dfrac{T^2}{4\pi^2} = \dfrac{(R+h)^3}{MG}$ ... (i)

Centripetal acceleration, $a = \dfrac{GM}{(R+h)^2}$

or $\quad \dfrac{(R+h)^2}{GM} = \dfrac{1}{a}$

or $\quad (R+h) = \dfrac{T^2}{4\pi^2} \times a$ [Using (i)]

$= (5.26 \times 10^3/2\pi)^2 \times 9.32$

$= 6.54 \times 10^6$ m

$\therefore \quad h = 6.54 \times 10^6 - R$

$= 6.54 \times 10^6 - 6.37 \times 10^6$

$= 0.17 \times 10^6$ m

$= 170$ km

33. (b): Given : $x = 4\sin\omega t + 3\sin(\omega t + \pi/3)$

Comparing it with the equation

$x = r_1\sin\omega t + r_2\sin(\omega t + \phi)$

we have, $r_1 = 4$ cm, $r_2 = 3$ cm and $\phi = \pi/3$

The amplitude of combination is

$$r = \sqrt{r_1^2 + r_2^2 + 2r_1r_2\cos\phi}$$

$$= \sqrt{4^2 + 3^2 + 2\times 4\times 3\times\cos\pi/3}$$

$$= \sqrt{37} \approx 6 \text{ cm}$$

34. (d): $P = 4.5 \times 10^5$ Pa, $\Delta Q = 800$ kJ

$V_1 = 0.5\text{ m}^3$, $V_2 = 2\text{ m}^3$

$\Delta W = P(V_2 - V_1) = 4.5 \times 10^5\,(2 - 0.5)$

$= 6.75 \times 10^5$ J

Change in internal energy

$\Delta U = \Delta Q - \Delta W = 800 \times 10^3 - 6.75 \times 10^5$

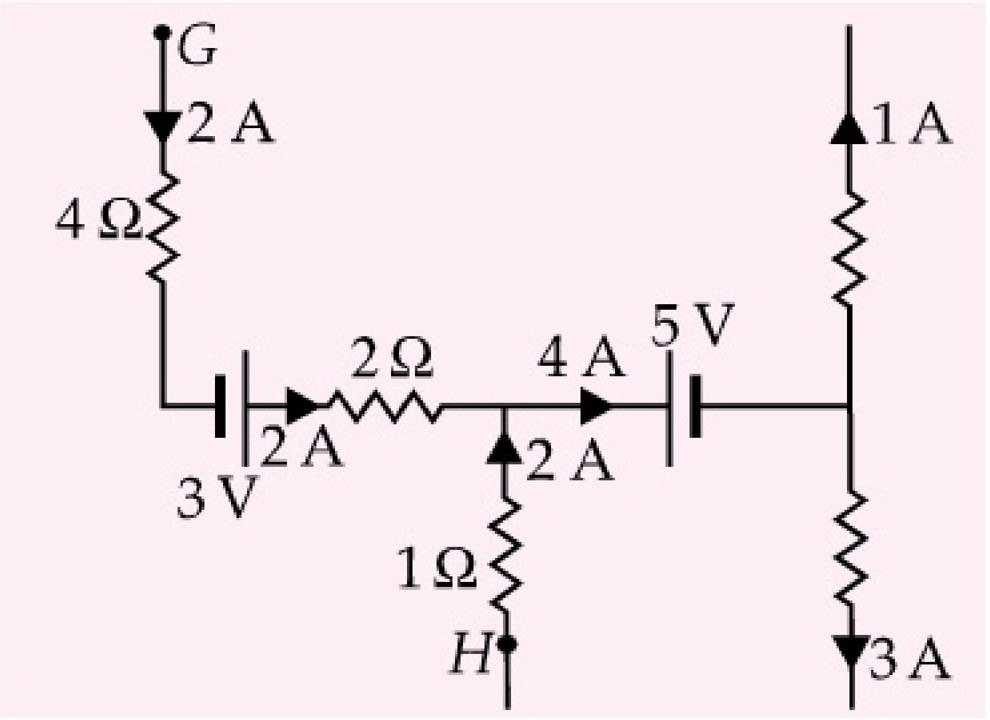
$= 1.25 \times 10^5$ J

35. (c) : There are five beats between A and B, therefore, the possible frequencies of A are

$512 \pm 5 = (517 \text{ or } 507)$ Hz.

When one prong of A is filed its frequency becomes greater than the original frequency. If we assume that the original frequency of A is 517 Hz, then on filing its frequency will be greater than 517 Hz. The beats between A and B will be more than 5. As it is given that the beats are increasing so it is only possible if frequency of A is 517 Hz.

36. (c) : The current distribution in a circuit is as shown in the figure.



Let V_G and V_H be the potentials at points G and H respectively.

$\therefore\ V_G - (2\text{ A})(4\,\Omega) + 3\text{ V} - (2\text{ A})(2\,\Omega) + (2\text{ A})(1\,\Omega) = V_H$

or $\ V_G - 8\text{ V} + 3\text{ V} - 4\text{ V} + 2\text{ V} = V_H$

or $\ V_G - V_H = 7$ V

37. (b): Here, both capacitors are in series

$$\therefore \quad \frac{1}{C_s} = \frac{1}{2} + \frac{1}{1} = \frac{3}{2}$$

or $\quad C_s = \dfrac{2}{3}$ F

$$\therefore \quad Q = C_sV = \frac{2}{3} \times 12 = 8 \text{ C}$$

So, potential difference across the capacitor of 2 F is

$$V_1 = \frac{Q}{C_1} = \frac{8}{2} = 4 \text{ V}$$

38. (b): The air pushed down by the wings of the parrot while flying will go out of the wire cage. Due to which the weight of wire cage will decrease.

39. (b): Here, $\upsilon = 50$ Hz, $R = 3\ \Omega$

$L = 25.48$ mH $= 25.48 \times 10^{-3}$ H

$C = 796\ \mu$F $= 796 \times 10^{-6}$ F

$$X_L = 2\pi\upsilon L$$

$$= 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3} = 8\ \Omega$$

$$X_C = \frac{1}{2\pi\upsilon C} = \frac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{-6}}$$

$$= 4\ \Omega$$

$\therefore$ Impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(3)^2 + (8-4)^2} = 5\ \Omega$$

40. (b): Total average energy density of electromagnetic wave is

$$\langle u \rangle = \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0}B_{\text{rms}}^2$$

$$= \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0}\left(\frac{E_{\text{rms}}^2}{c^2}\right) \quad \left(\because B_{\text{rms}} = \frac{E_{\text{rms}}}{c}\right)$$

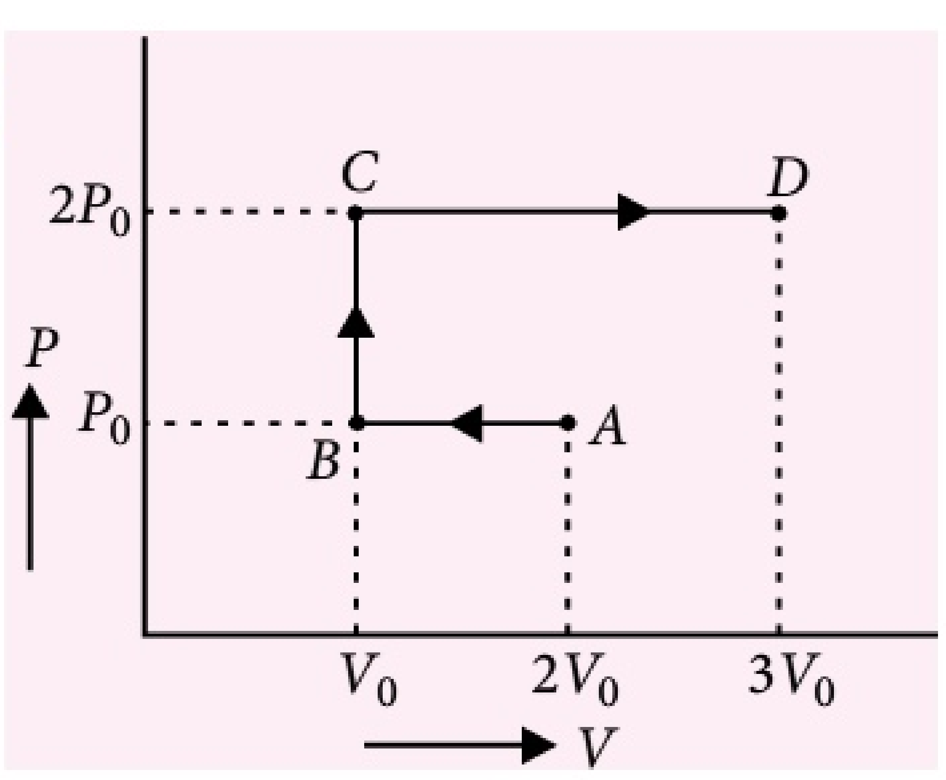
$$= \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2\mu_0}E_{\text{rms}}^2 \varepsilon_0\mu_0 \quad \left(\because \quad c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right)$$

$$= \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 + \frac{1}{2}\varepsilon_0 E_{\text{rms}}^2 = \varepsilon_0 E_{\text{rms}}^2$$

$= (8.85 \times 10^{-12}) \times (720)^2$

$= 4.58 \times 10^{-6}$ J m^{-3}

41. (c) :



As work done, $W = P\Delta V$

$\therefore \quad W_{AB} = -P_0V_0,$

$\quad W_{BC} = 0$

$\quad W_{CD} = 4P_0V_0$

Hence, $W_{ABCD} = W_{AB} + W_{BC} + W_{CD}$

$\quad = -P_0V_0 + 0 + 4P_0V_0 = 3P_0V_0$

42. (b): Let resistance of bulb filament at 0°C be R_0 and at a temperature θ°C its value be 200 Ω. Then, from $R_t = R_0(1 + \alpha t)$, we get,

$100 = R_0(1 + \alpha \times 100) = R_0(1 + 0.005 \times 100)$

$\quad = R_0(1.5) \quad$ ...(i)

and $200 = R_0(1 + \alpha \times \theta) = R_0(1 + 0.005 \times \theta)$

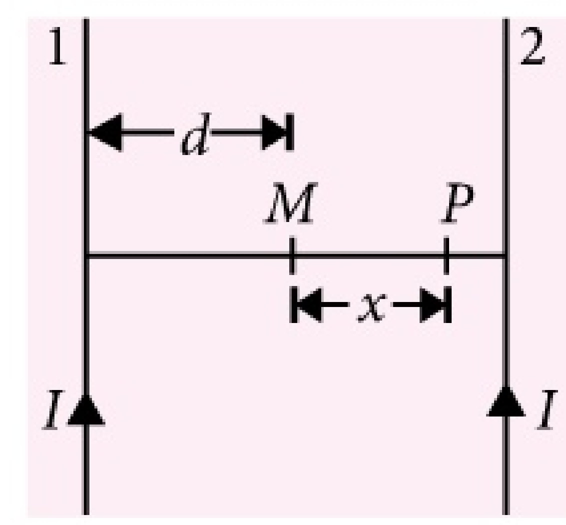
$\quad = R_0(1 + 0.005\theta)$ ...(ii)

Dividing eq. (ii) by eq. (i), we get

$$2 = \frac{1 + 0.005\theta}{1.5}$$

$$\Rightarrow \quad \theta = \frac{2}{0.005} = 400°\text{C}$$

43. (b): The magnetic field due to first wire



$$B_1 = \frac{\mu_0 I}{2\pi(d+x)}$$

Due to second wire

$$B_2 = \frac{\mu_0 I}{2\pi(d-x)}$$

Both the magnetic fields act in opposite directions.

$$\therefore \quad B = B_2 - B_1 = \frac{\mu_0 I}{2\pi}\left[\frac{1}{d-x} - \frac{1}{d+x}\right]$$

$$= \frac{\mu_0 I}{2\pi}\left[\frac{d+x-d+x}{d^2-x^2}\right] = \frac{\mu_0 Ix}{\pi(d^2-x^2)}$$

44. (d): Here, $u = 0$, $g = 10$ m s^{-2}, $t = 0.2$ s

As the lift is descending with $a = 12$ m s^{-2}

effective acceleration, $a' = g - a = 10 - 12$

$\quad = -2$ m s^{-2} (upwards)

From, $s = ut + \frac{1}{2}a't^2$

$$s = 0 + \frac{1}{2}(-2)(0.2)^2 = -0.04 \text{ m}$$

45. (c) : From horizontal position to lowest position, height through which the bob falls $= l$

$\quad = 1$ m

$\therefore$ At lowest position, $v = \sqrt{2lg}$

K.E. at lowest position $= \frac{1}{2}mv^2$

$$= \frac{1}{2}m(2lg) = mlg$$

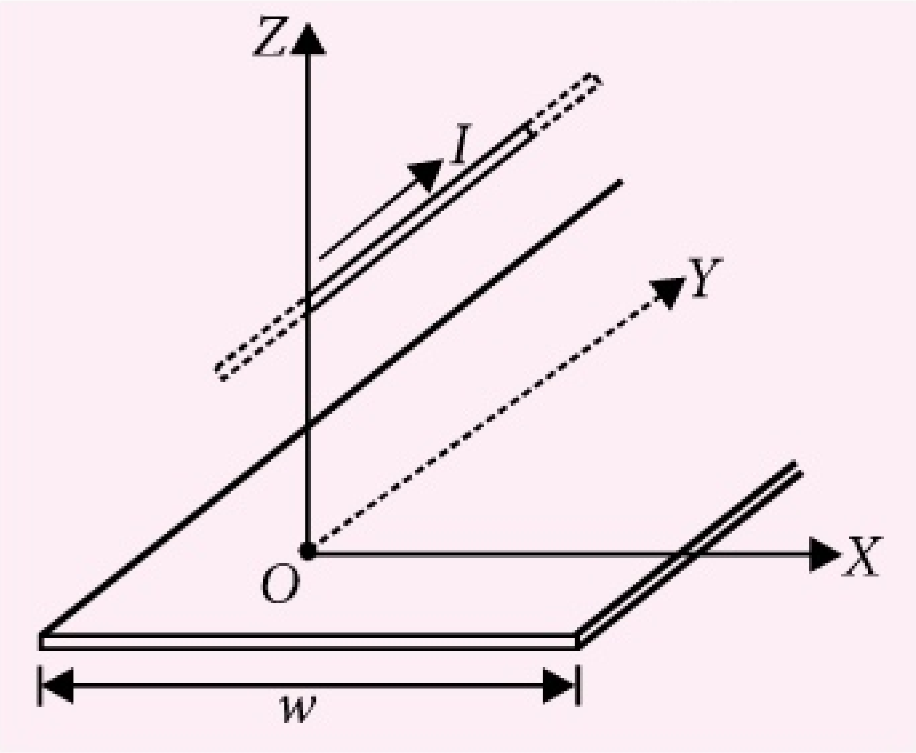
$= 10 \times 1 \times 9.8 = 98$ J.

Thought Provoking Problems

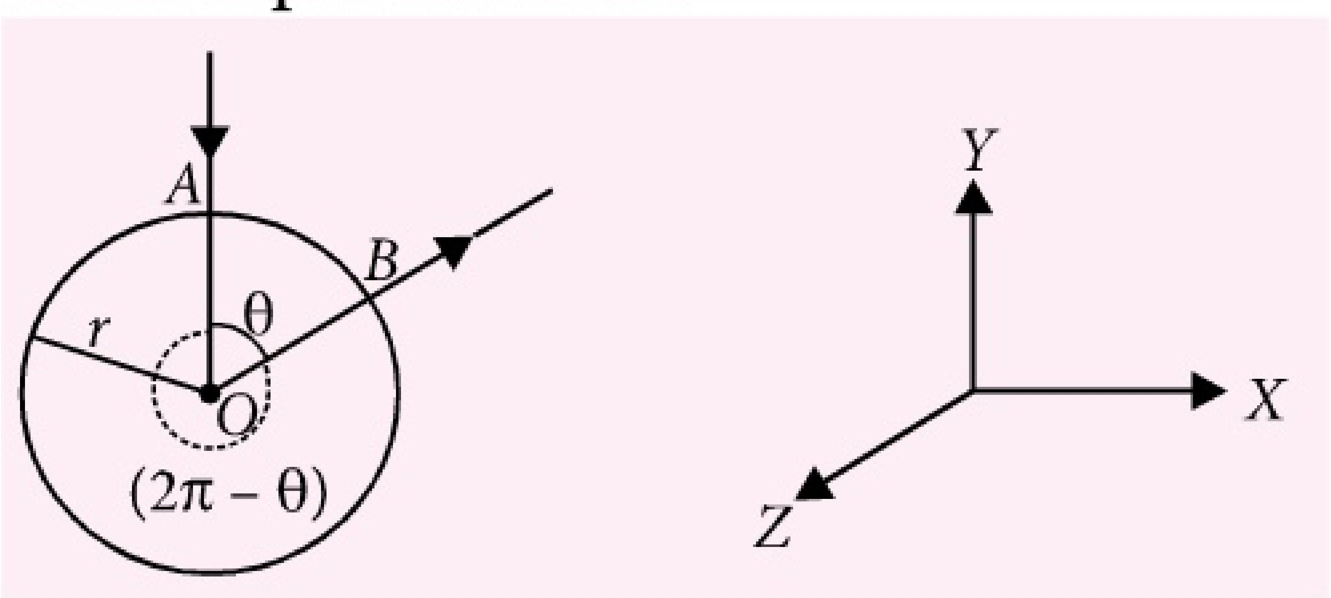
Magnetics

By : Prof. Rajinder Singh Randhawa*

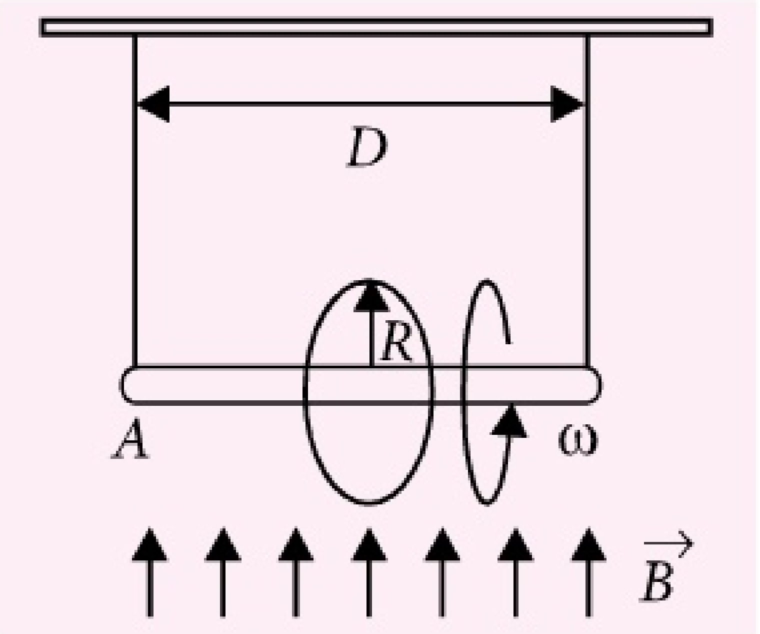
1. A conductor carries a current I parallel to a current strip of current per unit width j and width w, as shown in figure. Find an expression for the force per unit length on the conductor. Discuss the result when the width w approaches infinity.



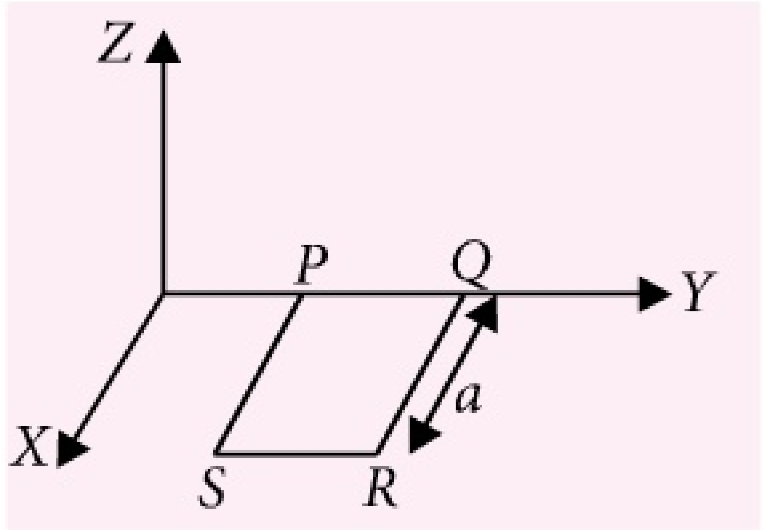
2. A battery is connected between two points A and B on the circumference of a uniform conducting ring of radius r and resistance R as shown in figure. One of the arc AB of the ring subtends angle θ at the centre. Show that the magnetic field at the centre of the coil is zero and independent of θ.



3. A ring of radius R having uniformly distributed charge Q is mounted on a rod suspended by two identical strings. The tension in strings in equilibrium is T_0. Now a vertical magnetic field is switched on and the ring is rotated at constant angular velocity ω. Find the maximum ω with which the ring can be rotated if the strings can withstand a maximum tension of $\frac{3T_0}{2}$.



4. A rectangular loop $PQRS$ made from a uniform wire has length a width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the Y-axis (see figure). Take the vertically upward direction as the Z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})B_0$ exists in the region. The loop is held in the X-Y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium.



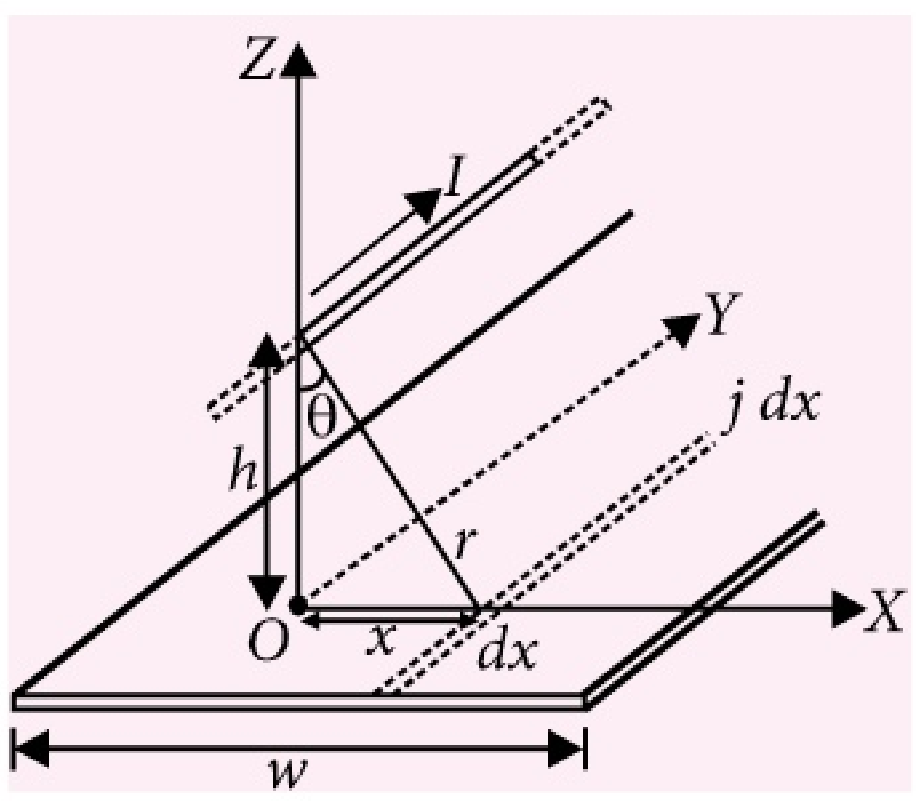
*Randhawa Institute of Physics, S.C.O. 208, First Fl., Sector-36D & S.C.O. 38, Second Fl., Sector-20C, Chandigarh

(a) What is the direction of the current I in PQ?
(b) Find the magnetic force on the arm RS.
(c) Find the expression for I in terms of B_0, a, b and m.

SOLUTIONS

1. Consider a small thickness dx, carrying current jdx. The attractive force per unit length is

$$\frac{dF}{L}=\frac{\mu_0 I(jdx)}{2\pi r}$$



A symmetrical segment at $-x$ exerts force of same magnitude, the x-components cancel and the resultant force per unit length is

$$\frac{dF}{L}=\frac{\mu_0 I(jdx)}{2\pi r}\cos\theta=\frac{\mu_0 Ijdx}{2\pi\sqrt{h^2+x^2}}\times\frac{h}{\sqrt{h^2+x^2}}$$

Integrating over half width of strip,

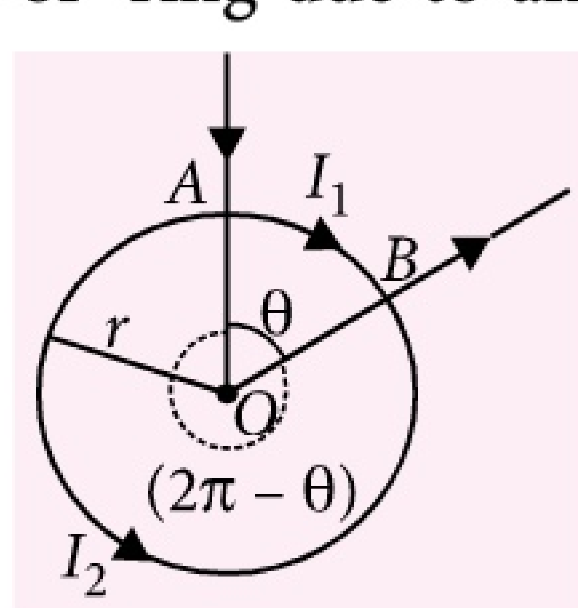
$$\frac{F}{L}=\frac{\mu_0 Ijh}{2\pi}2\int_0^{w/2}\frac{dx}{h^2+x^2}$$

$$\therefore\quad \frac{F}{L}=\frac{\mu_0 Ij}{\pi}\tan^{-1}\left(\frac{w}{2h}\right)$$

When $w\to\infty$, $\dfrac{F}{L}=\dfrac{\mu_0 Ij}{2}$

The force is attractive as expected for parallel currents.

2. Magnetic field at the centre of ring due to an arc is



$$B=\frac{\mu_0 I}{2r}\times\frac{\theta}{2\pi}$$

$\therefore$ Magnetic field due to smaller arc is

$$\vec{B}_1=\frac{\mu_0 I_1}{2r}\times\frac{\theta}{2\pi}(-\hat{k})$$

Magnetic field due to larger arc is

$$\vec{B}_2=\left[-\frac{\mu_0 I_2}{2r}\times\frac{(2\pi-\theta)}{2\pi}\right](+\hat{k})$$

Resultant magnetic field is

$$\vec{B}_R=\left[-\frac{\mu_0 I_1\theta}{4\pi r}+\frac{\mu_0 I_2(2\pi-\theta)}{4\pi r}\right](+\hat{k})\qquad ...(i)$$

Two arcs form a parallel combination of resistors

$\therefore\quad I_1R_1=I_2R_2$, where R_1 and R_2 are resistances of respective segments. As the wire is uniform,

$$\frac{R_1}{R_2}=\frac{L_1}{L_2}=\frac{r\theta}{r(2\pi-\theta)}\quad\therefore\quad\frac{I_2}{I_1}=\frac{\theta}{(2\pi-\theta)}$$

$$\Rightarrow\quad I_2=\frac{I_1\theta}{(2\pi-\theta)}\qquad ...(ii)$$

Putting (ii) in (i), we get

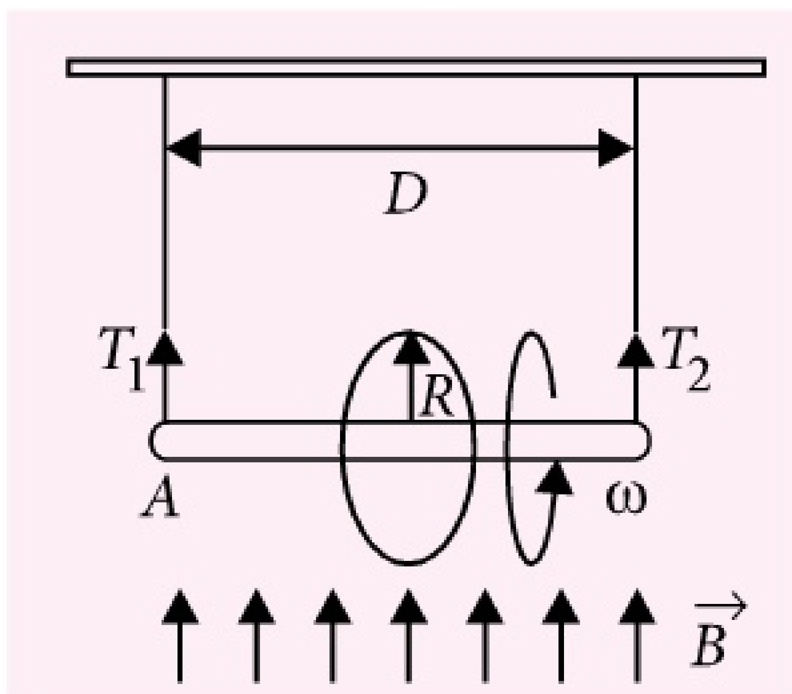
$$\vec{B}_R=\left[-\frac{\mu_0 I_1\theta}{4\pi r}+\frac{\mu_0 I_1\theta}{4\pi r}\right]\hat{k}=0$$

Hence the field is independent of θ.

3. In equilibrium, $2T_0 = mg$ or $T_0 = \dfrac{mg}{2}$ ... (i)

Magnetic moment, $M = IA = \left(\dfrac{\omega Q}{2\pi}\right)(\pi R^2)$

$\therefore \quad \tau = MB\sin 90° = \dfrac{\omega BQR^2}{2}$



For translational equilibrium of ring in vertical direction, $T_1 + T_2 = mg$ ... (ii)
For rotational equilibrium,

$$(T_1 - T_2)\frac{D}{2} = \tau = \frac{\omega BQR^2}{2} \qquad (\because\ T_1 > T_2)$$

or $\quad T_1 - T_2 = \dfrac{\omega BQR^2}{D}$ ...(iii)

Solving (ii) and (iii), we get

$$T_1 = \frac{mg}{2} + \frac{\omega BQR^2}{2D}$$

As $T_1 > T_2$ and maximum value of T_1 can be $\dfrac{3T_0}{2}$,
we have,

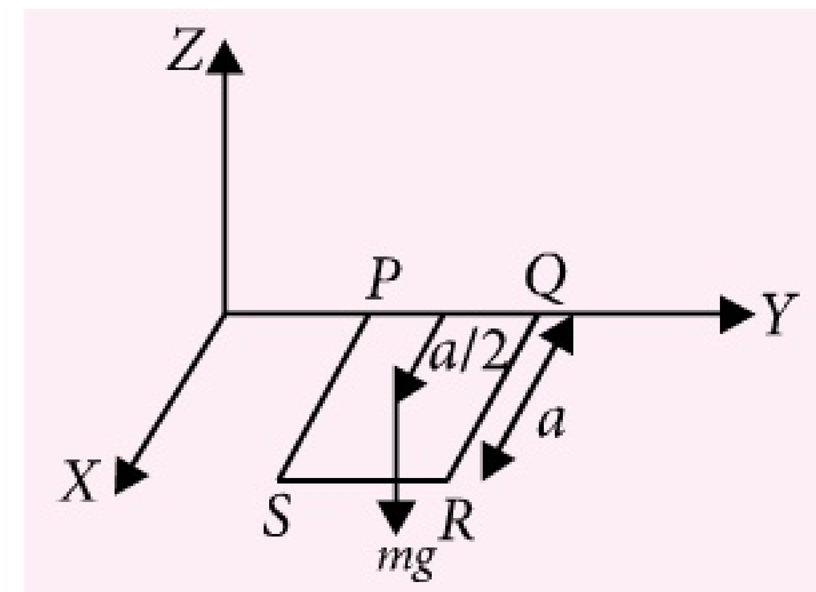
$$\frac{3T_0}{2} = T_0 + \frac{\omega_{max} BQR^2}{2D} \qquad \text{(Using (i))}$$

$$\therefore \quad \omega_{max} = \frac{DT_0}{BQR^2}$$

4. (a) : Torque due to weight of coil

$$\vec{\tau}_W = \left(\frac{a}{2}\hat{i}\right)(-mg\,\hat{k}) = mg\frac{a}{2}(\hat{j})$$

For the equilibrium of loop, torque on it must be along negative Y-axis.



Let the magnetic moment of loop be $M\hat{k}$

$\therefore$ Torque due to magnetic force,

$$\vec{\tau}_B = \vec{M} \times \vec{B} = M\hat{k} \times (3\hat{i} + 4\hat{k})B_0 = 3MB_0\,\hat{j}$$

$$\vec{\tau}_B = 3I(ab)B_0\,\hat{j}$$

If $\vec{\tau}_B$ is to be in negative direction, $\vec{M}$ must point downward. So the current in the coil must be from P to Q.

(b) Force acting on arm RS

$$= I(\vec{l} \times \vec{B}) = I[(-b\,\hat{j}) \times (3\hat{i} + 4\hat{k})B_0]$$

$$= IB_0 b(3\hat{k} - 4\hat{i})$$

(c) In equilibrium,

$$\vec{\tau}_W + \vec{\tau}_B = 0$$

$$\therefore \quad 3\,abIB_0 = mg\frac{a}{2}$$

or $\quad I = \dfrac{mg}{6B_0 b}$

CORE CONCEPT on

Heat Dissipation In Capacitive Circuits

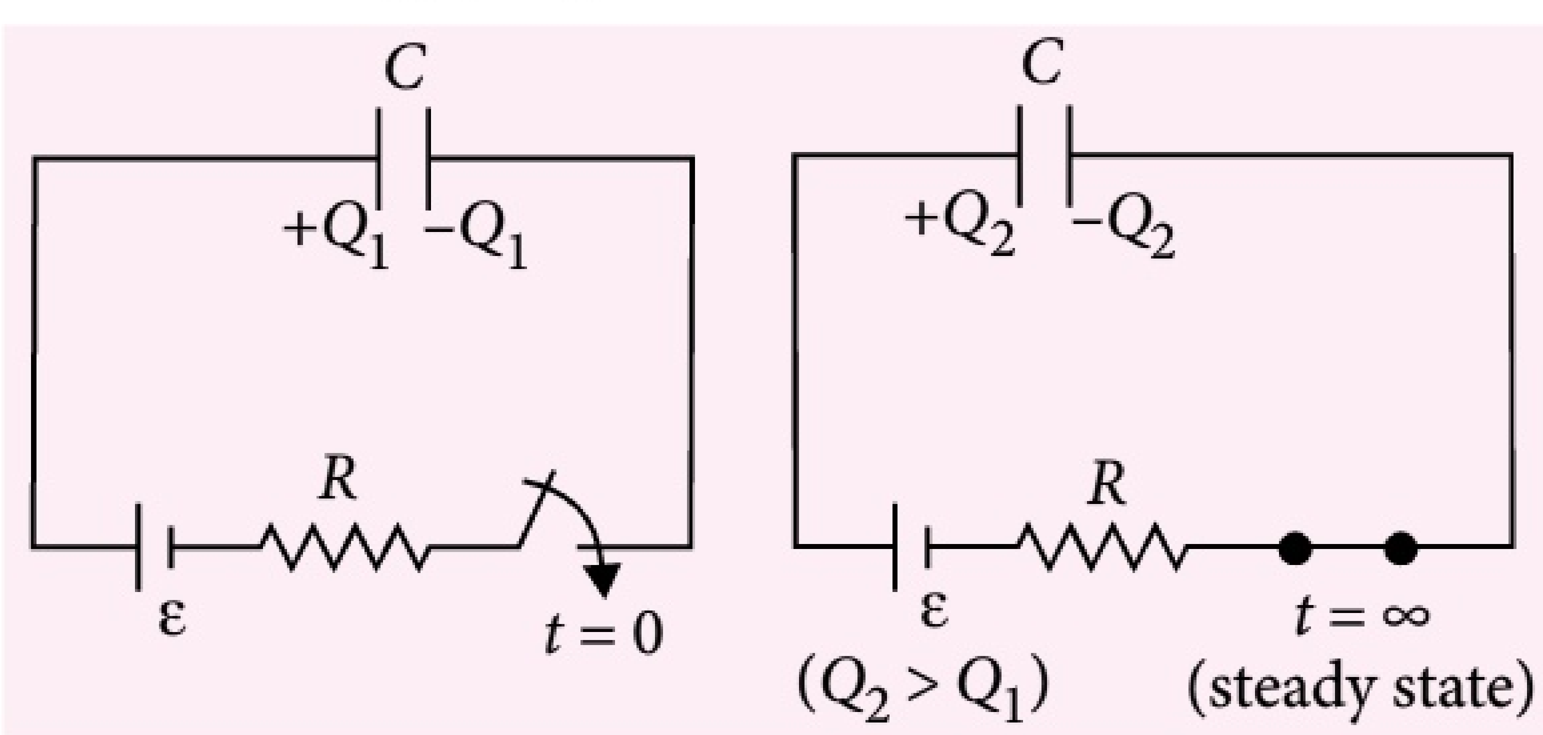
In the following topic, we discuss the energy loss in charging/discharging of a capacitor qualitatively as well as quantitatively in detail.

We take a charged capacitor of capacitance C with initial charge Q_1 and connect it to a cell of emf ε and resistance R in series.

For a case study, we take ε such that $C\varepsilon > Q_1$

where $C\varepsilon$ = Final steady state charge on capacitor

$= Q_2$ (say).



Therefore charge flown through the battery is,

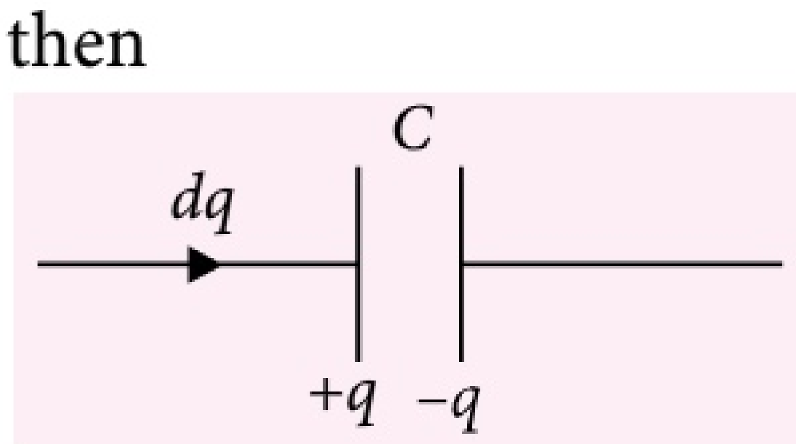
$$\Delta q = Q_2 - Q_1$$

$\therefore$ Work done by the battery is

$$W_{bb} = \Delta q\varepsilon = (Q_2 - Q_1)\varepsilon$$

$$= (Q_2 - Q_1)\left(\frac{Q_2}{C}\right) \qquad \text{...(i)}$$

During the transient duration (time in which the charge on capacitor changes). Let q be the instantaneous charge on the capacitor at any general time t (> 0), then



In a short duration dt, let dq amount of charge flows further across the terminals of the capacitor, therefore work done on capacitor which gets stored in the form of electrostatic field energy (also known as electrostatic potential energy) inside the capacitor is

$dW_{\text{on capacitor}} = dU = dq\left(\frac{q}{C}\right)$; where $\frac{q}{C}$ is the instantaneous potential difference across the terminals of capacitor.

$\therefore$ Change in potential energy of the capacitor,

$$\Delta U = \int dU = \int_{Q_1}^{Q_2} \frac{q}{C}\,dq = \frac{1}{2C}(Q_2^2 - Q_1^2)$$

$$= \left(\frac{Q_2 - Q_1}{C}\right)\left(\frac{Q_1 + Q_2}{2}\right) \qquad \text{...(ii)}$$

Clearly, $W_{bb} \neq \Delta U$ [From (i) and (ii)]

rather, $W_{bb} > \Delta U$.

So, what happened to the extra amount of the work done by the battery?

Now, we obviously understand that R is a resistor across which heat would be generated. Hence they are different.

$\therefore$ Heat generated,

$$H = W_{bb} - \Delta U$$

$$= (Q_2 - Q_1)\frac{Q_2}{C} - \frac{(Q_2 - Q_1)}{C}\frac{(Q_1 + Q_2)}{2}$$

$$= \frac{(Q_2 - Q_1)^2}{2C}$$

where $Q_2 - Q_1$ = change in charge on the capacitor plate.

$$\therefore \quad H = \frac{(\Delta q)^2}{2C} \qquad \text{...(iii)}$$

Contributed By: Bishwajit Barnwal, Aakash Institute, Kolkata

where $\Delta q = Q_2 - Q_1$

Now, the interesting question that would arise is, if we replace the conductor along with resistor by a superconducting *i.e.*, resistanceless wire, won't any energy loss take place in such case?

The answer lies below.

The amazing thing is that the result obtained for heat loss has nothing to do with the resistance of the conducting wires since at steady state, in any case, current does not flow across the terminals of the capacitor, hence the potential difference across the capacitor would still be equal to the emf of the cell, irrespective of the value of R.

So, how do we explain this energy loss now?

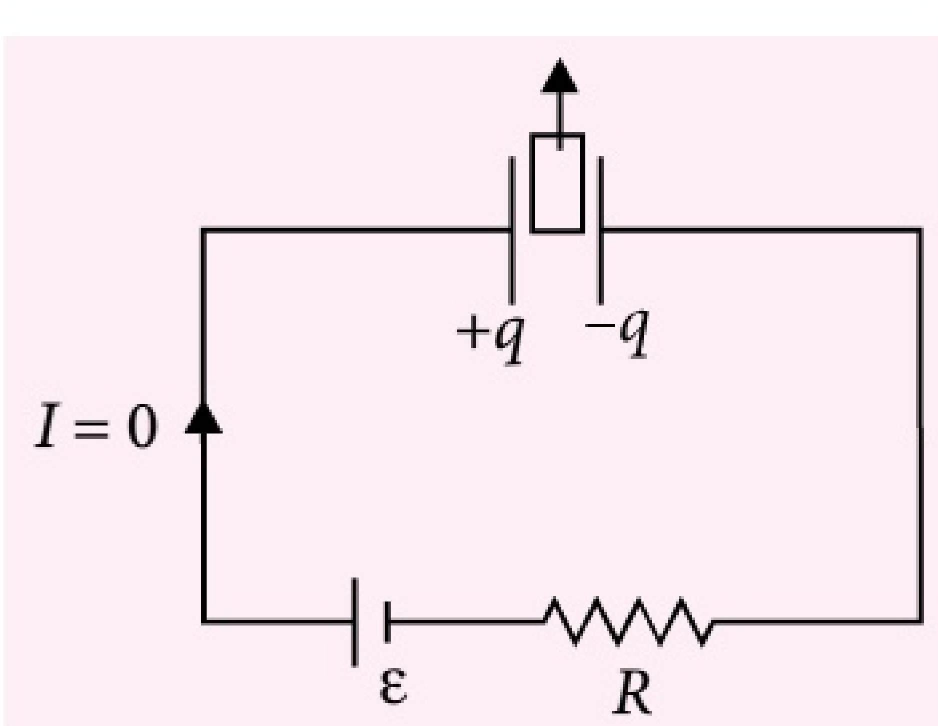
The possibilities of this energy loss is discussed below.

1. The electrons that are pulled from one plate of the capacitor to the other by the cell are accelerated in its due course of motion along the length of wire. As accelerated charges emit electromagnetic radiations, this is one possible source of loss.
2. The electrons that are accelerated, when they reach the other plate of the capacitor, which is fixed, undergo perfectly inelastic collision, since they are deposited and become bound charges there. This perfectly inelastic collision is a major source of energy loss.

So, can we conclude that whenever there is a change in charge on the capacitor, energy would be lost?

No ! we cannot !!

Consider the situation where a capacitor with a dielectric slab is connected to a cell and now the slab is being pulled out quasi-statically (slowly).



Since the process is slow, change in charge is at a very slow rate, *i.e.*, instantaneous current is zero at all instants. Hence the charges are not accelerated at all. Hence, no radiation or loss of energy in collision !

Though result for loss of energy (heat);

$$H = \frac{(\Delta q)^2}{2C} \qquad \text{[From equation (iii)]}$$

has been derived for a single capacitor, it is equally valid for multiple capacitors circuit where the total loss of energy is

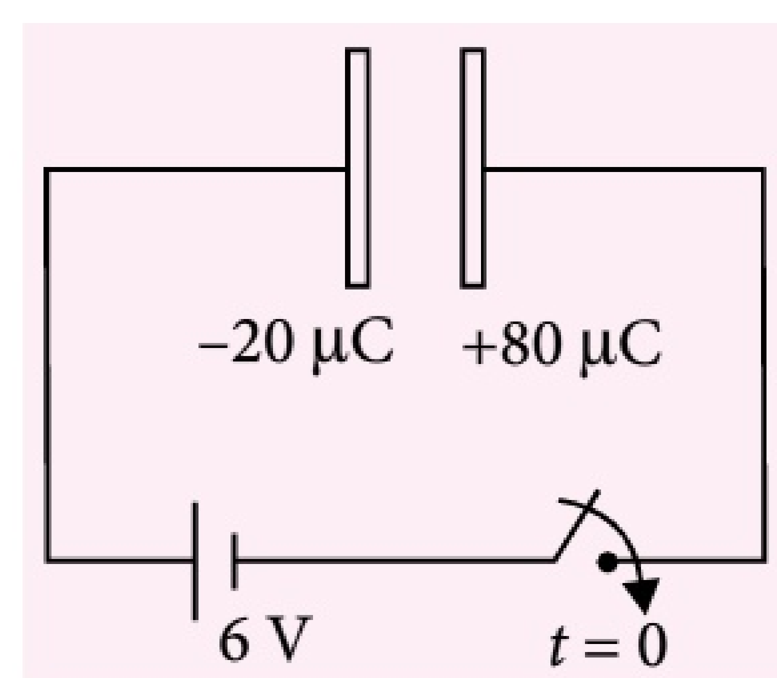
$$H = \frac{(\Delta q_1)^2}{2C_1} + \frac{(\Delta q_2)^2}{2C_2} + \ldots$$

$$\Rightarrow \quad H = \sum_{i=1}^{n} \frac{(\Delta q_i)^2}{2C_i}$$

where Δq_i = change in charge on the i^{th} capacitor of capacitance C_i.

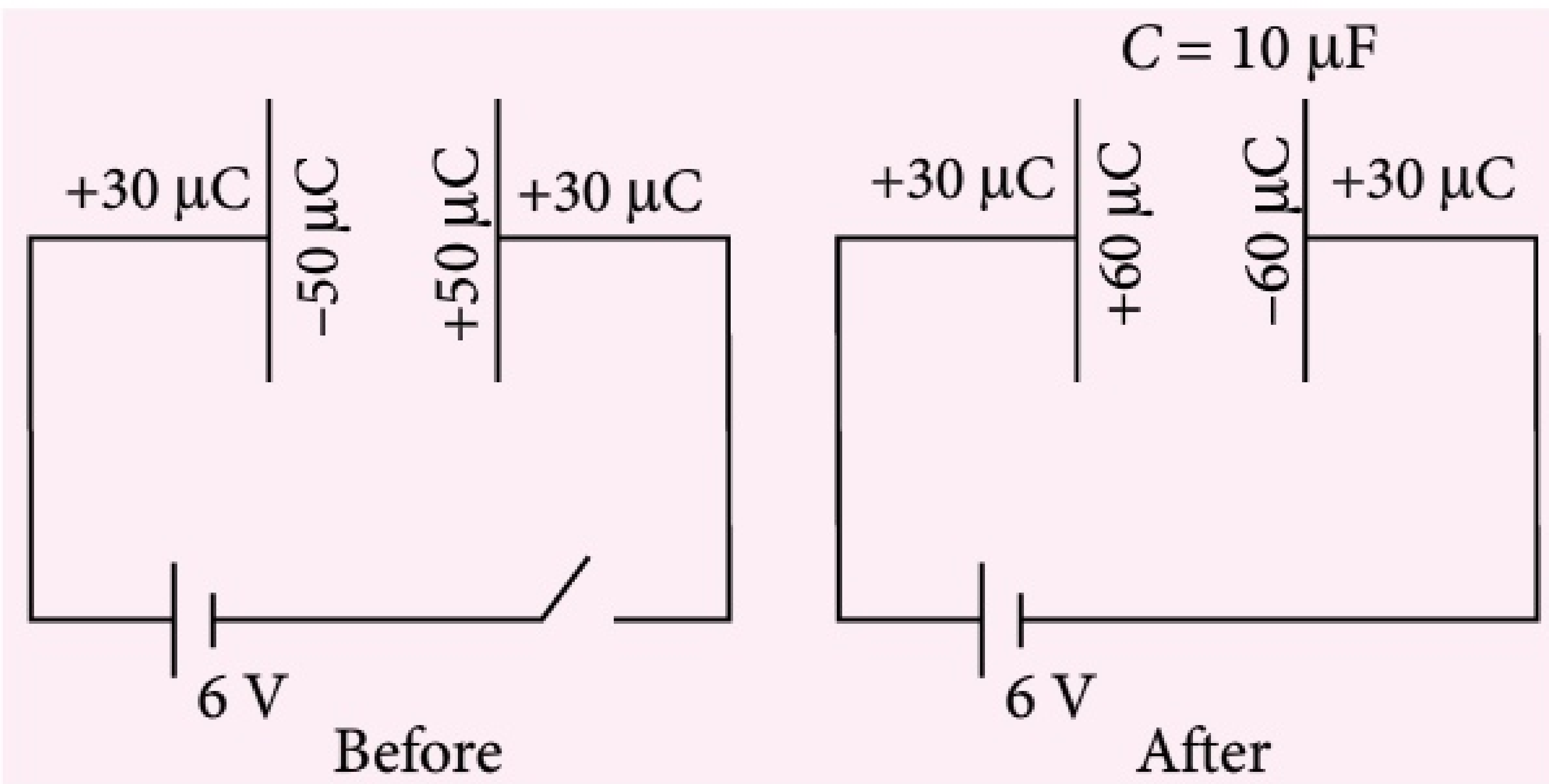
APPLICATIONS

1. Two parallel plates having charges – 20 μC and + 80 μC are connected to a cell of emf 6 V as shown.



The capacitance of the parallel plate capacitor is 10 μF. Find heat generated till steady state is attained.

Sol. The charge distribution on the four faces of the two plates, before and after closing the switch are :



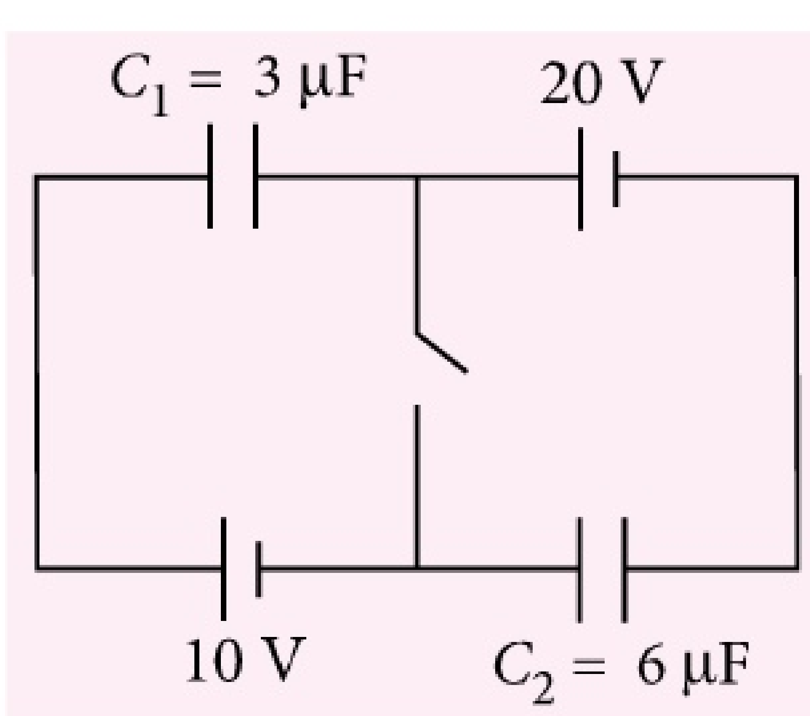
∴ Charge flown (change in charge on the capacitor),

$\Delta q = (60 - (-50))\ \mu C = 110\ \mu C$

∴ Heat generated,

$$H = \frac{(\Delta q)^2}{2C} = \frac{110 \times 110}{2 \times 10} = 605\ \mu J$$

2. In the following diagram, find the heat dissipated after closing the switch.

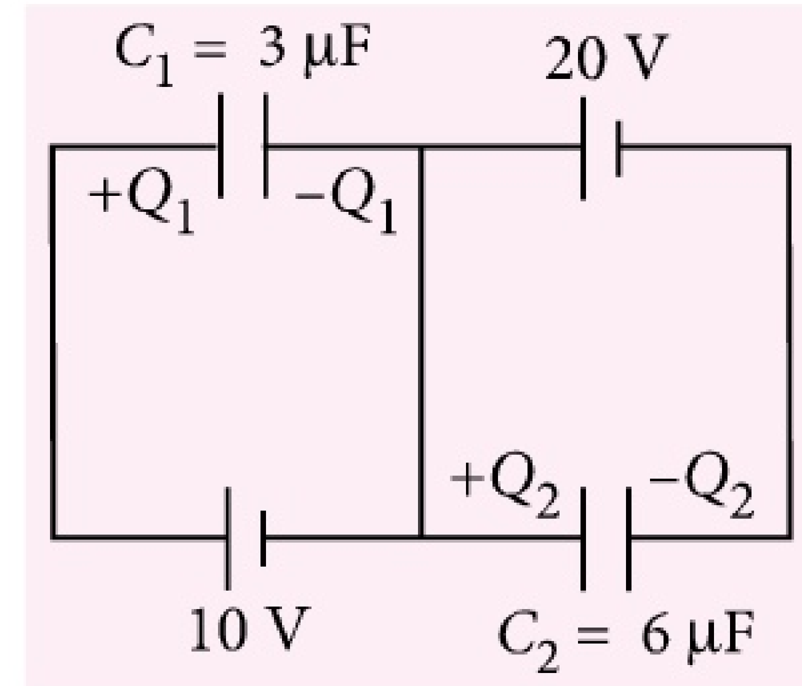


Sol. Before closing the switch, the capacitors C_1 and C_2 are in series connected across a net emf of $\varepsilon_{loop} = 20 - 10 = 10$ V.

∴ Charge on each capacitor is

$$q_1 = q_2 = \left(\frac{C_1C_2}{C_1 + C_2}\right)\varepsilon_{loop} = 20\ \mu C$$

After closing,



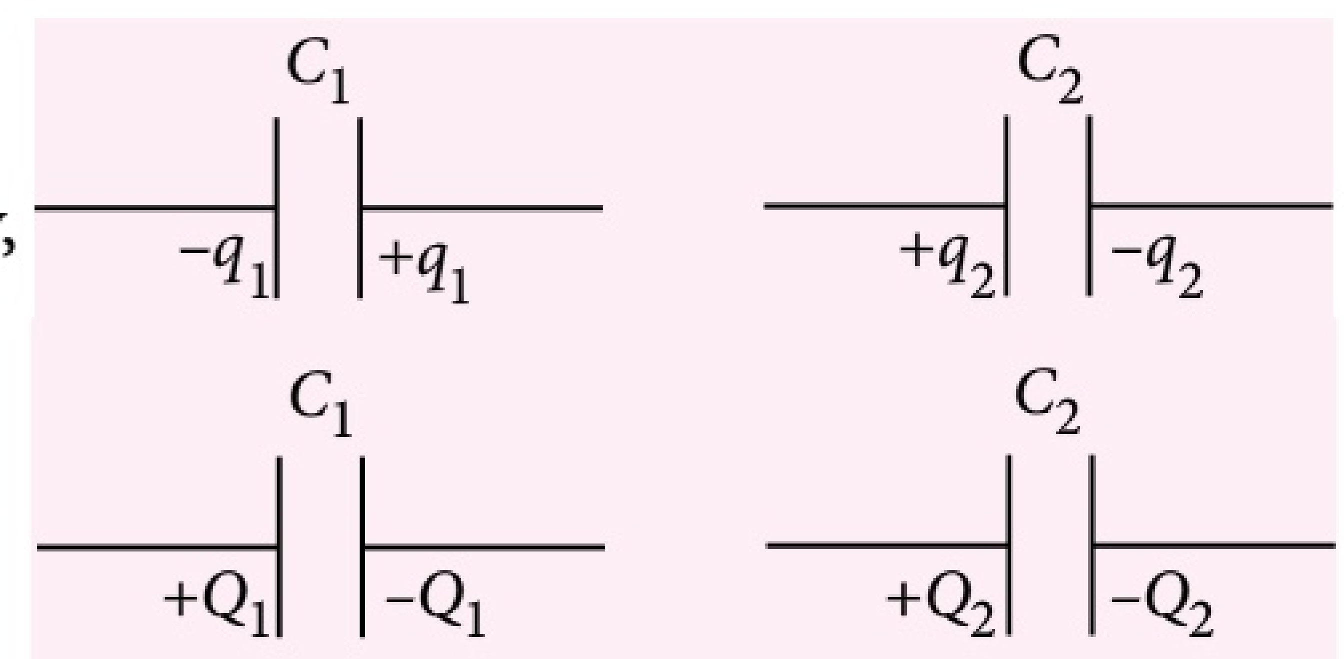
Clearly, now potential difference across C_1 is 10 V and across C_2 is 20 V, hence final charges on them, respectively are

$Q_1 = 3 \times 10 = 30\ \mu C$

$Q_2 = 20 \times 6 = 120\ \mu C$

Initially,

Finally,



∴ Change in charge on both capacitors are,

$\Delta q_1 = Q_1 - (-q_1) = Q_1 + q_1 = 50\ \mu C$

$\Delta q_2 = Q_2 - q_2 = 100\ \mu C$

∴ Heat generated,

$$H = \frac{(\Delta q_1)^2}{2C_1} + \frac{(\Delta q_2)^2}{2C_2} = \frac{(50)^2}{2 \times 3} + \frac{(100)^2}{2 \times 6}\ \mu J$$

$$= 1250\ \mu J$$

YOU ASKED WE ANSWERED

Do you have a question that you just can't get answered?
Use the vast expertise of our mtg team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.
The best questions and their solutions will be printed in this column each month.

Q1. In Judo, a weaker and smaller fighter who understands physics can defeat a stronger and larger fighter who does not. This fact is demonstrated by the basic "hip throw", in which a fighter rotates the fighter opponent around his hip and if the throw is successful on to the mat. Without the proper use of physics, the throw requires considerable strength and can easily fail. What is the advantage offered by physics?

– Samyajyoti Das, Tripura

Ans. This is a question for a judo master. Judo is a martial art that demand an intuitive understanding of the physics of force, torques, stability and rotational motion. The grace that each throw requires is not easily conveyed, by each throw can be broken into components that can be examined in terms of classical physics. In judo the main goal is to overcome your opponent's stability. The skill lies in the anticipation of his movements and the timing of your response. The idea is to avoid forcing your opponents into a firm resistance to your throw that would pit your strength against his. A small but skilled judo player has a distinct advantage over a larger but unskilled opponent if the contest of strength is avoided. Probably the best example of this advantage is the basic hip throw, which is most effective against a taller and slower opponent. In the normal judo competition you face your opponent with your hands grasping the lapels or shoulders of his uniform. To execute the throw your step forward with your right foot to a point in between his feet, pulling him downward and toward your right. The throw works well if you have caught your opponent just as he has stepped forward with his right foot. He is still stable against a pull directly toward you, but is considerably less stable against a pull to your right because of the position of his feet. During your step forward you curve your body forward so that your head is at your opponent's shoulder level. Next you rapidly turn your left hip backward while pulling him into your right hip. This shoulder be the first body contact during the movement. If you continue the pull with your hands and turn to rear with your left hip until you are facing in the same direction as your opponent, he will be rotated over your right hip and into the mat. Please never try to practise anything like this unless your master teaches you and also only in his presence initially.

Q2. If outside on a dark night in the middle to high latitudes, one might be able to see an aurora. It may be several hundred kilometers high and several thousand kilometers long, stretching around Earth in an arc. However, it is less than 1 km thick. What produces this huge display and what makes it so thin?

– Samyajyoti Das, Tripura

Ans. The aurora borealis (the Northern Lights) occur when highly charged electrons from the solar wind interact with elements in the earth's atmosphere. Solar winds stream away from the sun at speeds of about 1 million miles per hour. When they reach the earth, some 40 hours after leaving the sun, they follow the lines of magnetic force generated by the earth's core and flow through the magnetosphere, a teardrop-shaped area of highly charged electrical and magnetic fields.

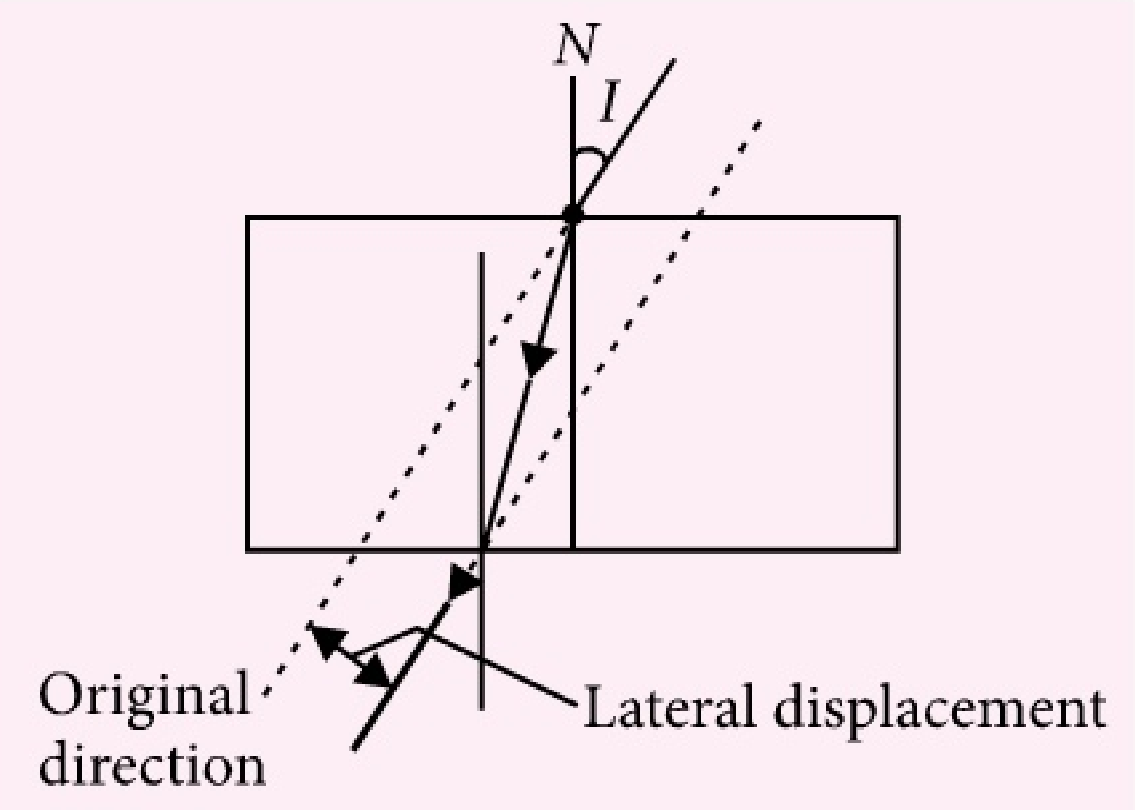
As the electrons enter the earth's upper atmosphere, they will encounter atoms of oxygen and nitrogen at altitudes from 20 to 200 miles above the earth's surface. The color of the aurora depends on which atom is struck, and altitude of the meeting.

The thickness of the aurora borealis depend on the thickness of the ionosphere.

Q3. Why is light dispersed by glass prism and not by parallel sided glass slab and light is laterally displaced by glass slab and not by prism?

– Santanu Chatterjee (WB)

Ans. A glass slab consists of two prisms.



The rays make the same angle i but takes a parallel path

Q4. In physics various physical quantities are often represented by the same symbol, like c for speed of light as well as for specific heat capacity, theta for temperature as well as angular displacement. Why is it so? Is this a flaw?

– Santanu Chatterjee (WB)

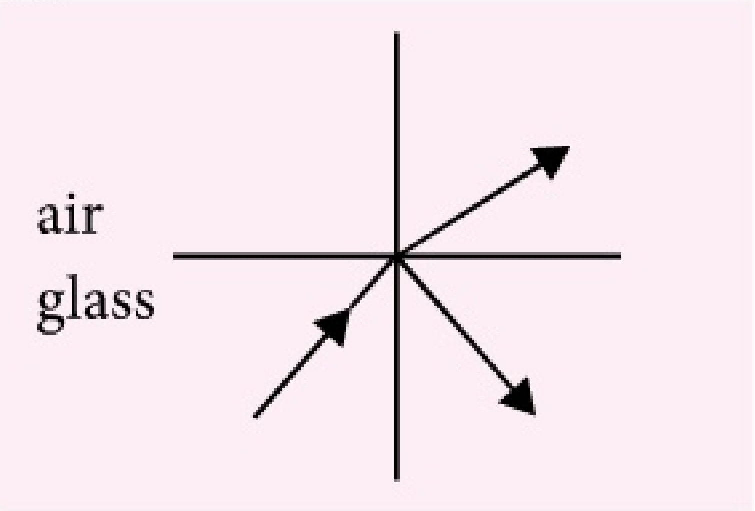
Ans. My great-grand father, myself and God Krishna share the same name. But you should know from the context that I have written this answer. If you had asked God he would have given a better answer. One will understand from the context the meaning of the symbol by practice.

Q5. Under what condition no ray emerges out of the prism?

– Shibasish Paul (Assam)

Ans. When there is total internal reflection.

(i) Usual reflection and refraction



Refractive index of glass with respect to air is

$$\mu_g = \frac{\sin i \ \text{(air)}}{\sin r \ \text{(glass)}}$$

μ is greater than one, 1.5 or 1.3 for example

$\sin i > \sin r$

$$\mu_g = \frac{1}{\sin C} \quad \text{if } i \text{ (air)} \geq 90°$$

For critical angle $i_{air} = 90°$

for any ray incident from denser to rarer medium, ray from glass to air, $\mu_g = \dfrac{1}{\sin C}$ is the condition for total internal reflection.

(This is used in periscopes because of clarity of images)

If the angle of incidence in the denser medium $\geq C$, the critical angle, the angle of refraction in air will be zero between the normal and the surfaces.

Q6. Which particle is negatively charged?

(a) Electron (b) Proton

(c) Neutron (d) None of these

– Suhail

Ans.

Particle	Symbol
Electron	e^-
Proton	p^+
Neutron	n°

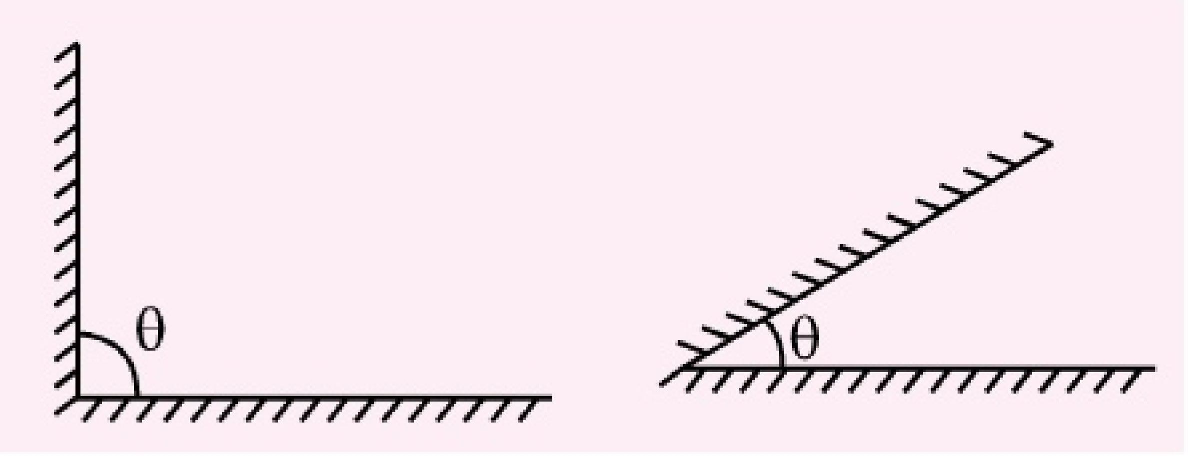
Neutron has no charge, so one can just write n.

Q7. What is the reason for only odd number of images being formed during reflection in plane mirror?

– Pramod Rao

Ans. What you are asking is the formation of images when two plane mirrors are kept at an angle to each other. Every image will produce its own image and finally when you find the last one coincides with another, then you stop. This causes the number to be odd (Find out using a problem).

Attention : If the mirrors are parallel, you get theoretically infinity. But beyond a certain number, even if the mirrors are very good, the images start fading.



CBSE BOARD 2015

CHAPTERWISE PRACTICE PAPER

SERIES-3

Magnetic Effects of Current and Magnetism

GENERAL INSTRUCTIONS

(i) All questions are compulsory.

(ii) There are 30 questions in total. Questions Nos. 1 to 8 are very short answer type questions and carry one mark each.

(iii) Questions Nos. 9 to 18 carry two marks each. Questions Nos. 19 to 27 carry three marks each and questions Nos. 28 to 30 carry five marks each.

(iv) One of the questions carrying three marks weightage is value based question.

(v) There is no overall choice. However, an internal choice has been provided in one question of two marks, one question of three marks and all three questions of five marks each weightage. You have to attempt only one of the choices in such questions.

(vi) Use of calculators is not permitted. However, you may use log tables if necessary.

1. How is the magnetic field inside a given solenoid made strong?

2. Which physical quantity has the unit Wb m^{-2}? Is it a scalar or a vector quantity?

3. A certain electric motor wires carry a current of 5 A perpendicular to magnetic field of 0.8 T. What is the force on each cm of the wires?

4. A current is set up in a long copper pipe. Is there a magnetic field : (a) inside (b) outside the pipe?

5. How does the intensity of magnetisation of paramagnetic material vary with increasing applied magnetic field?

6. A magnetic dipole is situated in the direction of a magnetic field. What is its potential energy? If it is rotated by 180°, then what amount of work will be done?

7. A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. What would be the ratio of the radii of circular path described by them?

8. A voltmeter, an ammeter and a resistance are connected in series with a lead accumulator. The voltmeter gives some deflection but the deflection of ammeter is almost zero. Explain why?

9. A charged particle is moving on a circular path of radius R in uniform magnetic field under the Lorentz force F. How much work is done by the force in one round? Is the momentum of the particle is changing ?

10. Write the relation for the force $\vec{F}$ acting on a charge q moving with a velocity $\vec{v}$ through a magnetic field $\vec{B}$ in vector notation. Using this relation, deduce the conditions under which this force will be (i) maximum (ii) minimum.

11. A current carrying circular loop is located in a uniform external magnetic field. If the loop is free to turn, what is its orientation of stable equilibrium? Show that in this orientation, the flux of the total field (external field + field produced by the loop) is maximum.

12. Magnetic field at the centre of a circular loop of area A carrying current I is B. What is the magnetic moment of this loop?

13. A current of one ampere is passed through a straight wire of length 2 m. Find the magnetic field at a point in air at a distance 3 m from one end of wire but lying on the axis of the wire.

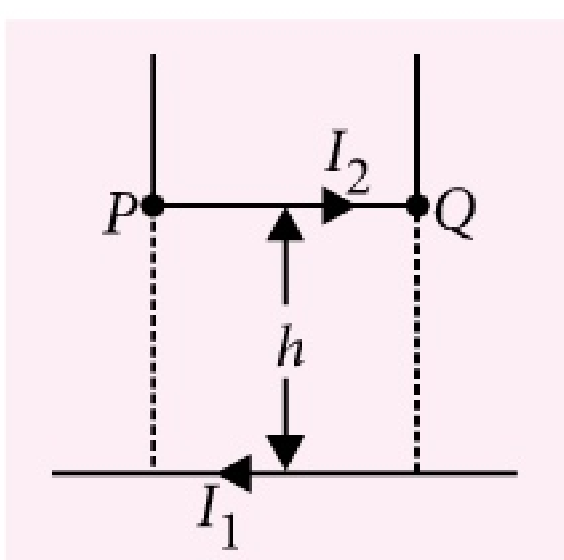
14. A loop of irregular shape carrying current is located in an external magnetic field. If the wire is flexible, why does it change to circular shape?

15. A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current is 8 A. Estimate the magnitude of $\vec{B}$ inside the solenoid near its centre.

16. Describe the motion of a charged particle in a cyclotron if the frequency of the radio frequency field were doubled.

OR

A long straight wire carrying current of 25 A rests on a table as shown in figure. Another wire PQ of length 1 m, mass 2.5 g carries the same current but in the opposite direction. The wire PQ is free to slide up and down. To what height will PQ rise?



17. An iron bar magnet is heated to 1000°C and then cooled in a magnetic field free space. Will it retain magnetism?

18. A closely wound solenoid of 2000 turns and area of cross section 1.6×10^{-4} m^2, carrying a current of 4 A is suspended through its centre allowing it to turn in a horizontal plane.

(a) What is the magnetic moment associated with the solenoid?

(b) What are the force and torque on the solenoid if a uniform horizontal magnetic field of 7.5×10^{-2} T is set up at an angle of 30° with the axis of the solenoid?

19. A toroid has a core (non ferromagnetic material) of inner radius 25 cm and outer radius 26 cm around which 3500 turns of wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid (b) inside the core of the toroid (c) in the empty space surrounded by the toroid?

20. On a field Rohit and Varun along with their friends were playing football. Suddenly, a player kicked out the ball outside the field and the ball reached an electric power substation. One of the players was planning to go and bring back the ball to the field but, Rohit and Varun stopped him, saying that it is dangerous to go to the substation as warmed up body can be caught by the strong magnetic field, which is created by high voltage current in conductors of the switchyard and that may result in death.

(a) Rohit and Varun are the student of class 12. Consider yourself as this friend and give your opinion about them.

(b) There is conductor of length L in the magnetic field $\vec{B}$, current I flows through the conductor. Find the force acting on the conductor ($|\vec{B}|$ = constant).

21. A circular coil of 16 turns and radius 10 cm carrying current of 0.75 A rests with its plane normal to an external field of magnitude 5×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of 2 s^{-1}. What is the moment of inertia of the coil about its axis of rotation?

OR

How can a moving coil galvanometer be converted into an ammeter? To increase the current sensitivity of a moving coil galvanometer by 50% its resistance is increased so that the new resistance becomes twice its initial resistance. By what factor does its voltage sensitivity change?

Draw the field lines due to an external magnetic field near a (i) diamagnetic, (ii) paramagnetic substances.

OR

State Ampere's circuital law. Using this law derive an expression for the magnetic field along the axis of an air-cored solenoid.

SOLUTIONS

1. Magnetic field inside a given solenoid can be made strong by inserting laminated iron core inside the solenoid.
2. Wb m^{-2} is the SI unit of magnetic field intensity B. It is a vector quantity.
3. Here, $I = 5$ A, $B = 0.8$ T, $l = 1$ cm $= 0.01$ m
Thus, $F = IlB = (5 \times 0.01 \times 0.8)$ N $= 0.04$ N
4. According to Ampere's circuital law,
$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$
where I is the current enclosed with the loop.
If r is distance of the point from the centre of the pipe,
(a) $B_{\text{inside}} = 0$, as $I = 0$
(b) $B_{\text{outside}} \propto \dfrac{1}{r}$, as $I \neq 0$
5. For low values of B, the intensity of magnetisation increases proportionally with B. For sufficiently high values of B, when all the atomic magnetic dipoles get aligned with B, intensity of magnetisation becomes independent of B.
6. Potential energy of dipole $= -MB\cos 0° = -MB$
Work done $= MB(\cos 0° - \cos 180°)$
$= MB(1 + 1)$
$= 2MB$
7. For given momentum of charged particle, radius of circular path depends on charge and magnetic field as
$$r = \frac{mv}{qB} \Rightarrow r = \frac{p}{qB}$$
$\therefore$ For given momentum,
$r_{\text{proton}} : r_{\text{deuteron}} = 1 : 1$
8. Voltmeter resistance being very high, when connected in series, it makes the effective resistance of the circuit very high. Due to it, the current in the circuit becomes extremely small. Since ammeter measures the current, hence the deflection of ammeter is almost zero. As voltmeter measures potential difference between the two points, it will show the reading due to voltage of the lead accumulator.
9. When a charged particle is moving on a circular path in a uniform magnetic field, the angle between force $\vec{F}$ due to magnetic field and small displacement $d\vec{r}$ is 90°. So work done, $dW = \vec{F}.d\vec{r} = F\, dr \cos 90° = 0$. Since the velocity of the particle is tangent to the circular path at a point, hence velocity or momentum is changing during circular motion.
10. $\vec{F} = q(\vec{v} \times \vec{B})$ or $|\vec{F}| = q|\vec{v} \times \vec{B}| = qvB\sin\theta$
(i) F will be maximum, when $\sin\theta = 1$ or $\theta = 90°$, *i.e.*, the charged particle is moving perpendicular to the direction of magnetic field.
(ii) F will be minimum, when $\sin\theta = 0$ or $\theta = 0°$ or $180°$ *i.e.*, the charged particle is moving parallel to the direction of magnetic field.
11. The current carrying circular loop behaves as a magnetic dipole of magnetic moment $\vec{M}$ acting perpendicular to its plane. The torque on the current loop of magnetic dipole moment M in the magnetic field B is
$\tau = MB \sin \alpha = IAB \sin \alpha$ $\qquad (\because M = AI)$
where α is the angle between $\vec{M}$ and $\vec{B}$. The system will be in stable equilibrium if torque is zero, which is so if $\alpha = 0°$. This is possible if $\vec{B}$ is parallel to $\vec{A}$. *i.e.* $\vec{B}$ is perpendicular to the plane of the loop.
In this orientation, the magnetic field produced by the loop is in the same direction as that of external field, both normal to the plane of loop. It is due to this fact, the magnetic flux due to total field is maximum.
12. Let R be the radius of the circular loop. Then,
$$A = \pi R^2 \quad \text{or} \quad R = \sqrt{A/\pi}$$

Draw the field lines due to an external magnetic field near a (i) diamagnetic, (ii) paramagnetic substances.

OR

State Ampere's circuital law. Using this law derive an expression for the magnetic field along the axis of an air-cored solenoid.

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In this orientation, the magnetic field produced by the loop is in the same direction as that of external field, both normal to the plane of loop. It is due to this fact, the magnetic flux due to total field is maximum.
12. Let R be the radius of the circular loop. Then,
$$A = \pi R^2 \quad \text{or} \quad R = \sqrt{A/\pi}$$

Magnetic field induction at the centre of circular loop carrying current I is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I}{R} \quad \text{or} \quad I = \frac{2BR}{\mu_0}$$

$\therefore$ Magnetic moment, $M = IA$

$$= \frac{2BR}{\mu_0}A = \frac{2BA}{\mu_0}\sqrt{\frac{A}{\pi}}$$

13. When a point P lies on the axis of wire in air, then $I\,d\vec{l}$ and $\vec{r}$ for each element of the straight wire are parallel. Therefore, $I\,d\vec{l} \times \vec{r} = 0$, so the magnetic field induction at point P is zero.

14. A loop of irregular shape carrying current when placed in an external magnetic field, assumes circular shape with its plane normal to the field in order to maximise magnetic flux linked with loop, since for a given perimeter, a circle encloses maximum area than any other shape.

15. Here, l = 80 cm = 0.80 m, $N = 5 \times 400 = 2000$, I = 8 A, D = 1.8 cm.
Magnitude of magnetic field induction at a point inside the solenoid is

$$B = \frac{\mu_0 NI}{l} = \frac{4\pi\times10^{-7}\times2000\times8}{0.80}$$

$= 8\pi \times 10^{-3}$ T $\approx 2.5 \times 10^{-2}$ T

16. Time period of revolution of charged particle in cyclotron, $T = \frac{2\pi m}{Bq}$

Frequency of revolution, $\upsilon = \frac{1}{T} = \frac{Bq}{2\pi m}$

As frequency of revolution is independent of radius and velocity of particle, so the motion of charged particle will remain unchanged.

OR

Here, I_1 = 25 A, I_2 = 25 A, l = 1 m,
m = 2.5g = 2.5×10^{-3} kg
In equilibrium position

$$mg = \frac{\mu_0}{4\pi}\frac{2I_1I_2l}{h}$$

or $$h = \frac{\mu_0}{4\pi}\frac{2I_1I_2l}{mg} = \frac{10^{-7}\times2\times25\times25\times1}{(2.5\times10^{-3})\times9.8}$$

$= 51 \times 10^{-4}$ m = 0.51 cm

17. Curie temperature of iron is 770°C. 1000°C is above the Curie temperature, therefore at 1000°C iron bar will lose its magnetism. Again when cooled in a magnetic field free space it will not retain magnetism.

18. Here, N = 2000, $A = 1.6 \times 10^{-4}$ m^2, I = 4 A
(a) $M = NIA = 2000 \times 4 \times 1.6 \times 10^{-4} = 1.28$ J T^{-1}
(b) Net force on the solenoid = 0
Torque, $\tau = MB\sin\theta$

$= 1.28 \times 7.5 \times 10^{-2} \times \frac{1}{2} = 4.8\times10^{-2}$ N m

19. Here, r_1 = 0.25 cm, r_2 = 0.26 m, N = 3500,
I = 11 A
(a) Outside the toroid, the magnetic field is zero.
(b) Inside the core of the toroid, the magnetic field induction, $B = \mu_0 nI$,
where n is the number of turns per unit length of toroid = N/l.
Here, mean length of toroid,

$$l = 2\pi\left(\frac{r_1+r_2}{2}\right)$$

$= \pi(r_1 + r_2) = \pi(0.25 + 0.26)$
$= \pi \times 0.51$ m

So, $$B = \frac{\mu_0 NI}{l} = \frac{(4\pi\times10^{-7})\times3500\times11}{\pi\times0.51}$$

$= 3.02 \times 10^{-2}$ T

(c) In the empty space surrounded by toroid, the magnetic field is zero.

20. (a) Rohit and Varun displayed a knowledgeable character having sound technical knowledge with an explaining ability to convince other even at very young age. So, we can say that both of them handled the situation ideally, with there presence of mind and helping behaviour.
(b) Force acting on the conductor in a constant magnetic field
$\vec{F} = I\vec{L}\times\vec{B} = ILB\sin\theta$

where,
θ = angle between current element in the direction of current and magnetic field.
L = length of the conductor
B = magnetic field
I = current through the conductor in constant magnetic field B

21. Here, $N = 16$, $r = 10$ cm $= 0.1$ m, $I = 0.75$ A, $B = 5 \times 10^{-2}$ T, $\upsilon = 2\ \mathrm{s}^{-1}$

$$M = NIA = NI\pi r^2 = 16 \times 0.75 \times \frac{22}{7}(0.1)^2$$
$$= 0.377\ \mathrm{J\ T^{-1}}$$

As $\upsilon = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$ where I is the moment of inertia of the coil.

$$\therefore \quad \upsilon^2 = \frac{MB}{4\pi^2 I}$$

or $$I = \frac{MB}{4\pi^2\upsilon^2} = \frac{0.377 \times 5 \times 10^{-2}}{4 \times \left(\frac{22}{7}\right)^2 \times 2^2}$$
$$= 1.2 \times 10^{-4}\ \mathrm{kg\ m^2}$$

OR

A galvanometer can be converted into an ammeter by connecting a small resistance (called shunt) in parallel with it.

Here, $I_s' = I_s + \frac{50}{100}I_s = \frac{150}{100}I_s = \frac{3}{2}I_s$
$R' = 2R$

Initial voltage sensitivity, $V_s = \frac{I_s}{R}$

New voltage sensitivity, $V_s' = \frac{I_s'}{R'} = \frac{\frac{3}{2}I_s}{2R} = \frac{3}{4}V_s$

Percentage decrease in voltage sensitivity

$$= \frac{V_s - V_s'}{V_s} \times 100 = \left(1 - \frac{V_s'}{V_s}\right) \times 100$$
$$= \left(1 - \frac{3}{4}\right) \times 100 = 25\%$$

22. Here, $m = 8.4$ kg, $\upsilon = 50\ \mathrm{s}^{-1}$, $\rho = 7200\ \mathrm{kg\ m^{-3}}$

$$\text{Volume} = \frac{m}{\rho} = \frac{8.4}{7200} = 1.167 \times 10^{-3}\ \mathrm{m^3}$$

Energy dissipated in 30 minutes $= 3.2 \times 10^4$ J
Energy dissipated per second

$$= \frac{3.2 \times 10^4}{30 \times 60} = 17.78\ \mathrm{J\ s^{-1}}$$

Energy dissipated per unit volume per cycle

$$= \frac{17.78}{1.167 \times 10^{-3} \times 50} = 304.7\ \mathrm{J\ m^{-3}\ cycle^{-1}}$$

23. Here, $\mu_r = 400$, $I = 2$ A, $n = 1000$ per metre

(i) $H = nI = 1000 \times 2 = 2 \times 10^3\ \mathrm{A\ m^{-1}}$

(ii) $B = \mu H = \mu_0\mu_r H$
$= 4\pi \times 10^{-7} \times 400 \times (2 \times 10^3) = 1.0$ T

(iii) The magnetising current I_m is the additional current that needs to be passed through the windings of the solenoid in the absence of the core, which would produce a B value as in the presence of the core.

Thus, $B = \mu_0 n(I + I_m)$
$1 = 4\pi \times 10^{-7} \times 1000 \times (2 + I_m)$

$$I_m = \frac{1}{4\pi \times 10^{-4}} - 2 = 796 - 2 = 794\ \mathrm{A}$$

24. According to Biot-Savart's law, the magnetic field due to a small circular current element of length Δl at centre O is

$$\Delta B = \frac{\mu_0 I \Delta l \sin 90^\circ}{4\pi r^2}$$

while the magnetic field due to straight portions will be zero (since $\sin\theta = 0$)

$\therefore$ Total magnetic field at centre O

$$B = \frac{\mu_0 I}{4\pi r^2}\Sigma \Delta l = \frac{\mu_0 I}{4\pi r^2} r\theta \quad (\because \ \Sigma\Delta l = r\theta)$$
$$= \frac{\mu_0 I\theta}{4\pi r}$$

25. Here, $r = 0.51$ Å $= 0.51 \times 10^{-10}$ m,
$v = 2 \times 10^5\ \mathrm{m\ s^{-1}}$

Time period $T = \frac{2\pi r}{v}$

(a) Electric current

$$I = \frac{e}{T} = \frac{e}{\left(\frac{2\pi r}{v}\right)} = \frac{ev}{2\pi r}$$

$$I = \frac{1.6\times10^{-19}\times2\times10^{5}}{2\times3.14\times0.51\times10^{-10}} = 9.99\times10^{-5}\text{ A}$$

(b) $B = \dfrac{\mu_0 I}{2r}$

$$= \frac{4\pi\times10^{-7}}{2\times0.51\times10^{-10}}\times9.99\times10^{-5}$$

$B = 1.23$ T

(c) Magnetic moment $M = IA$

$= 9.99\times10^{-5}\times(\pi\times r^2)$

$= 9.99\times10^{-5}\times3.14\times0.51\times0.51\times10^{-20}$

$= 8.16\times10^{-25}$ A m^2

26. Here, number of dipoles, $n = 2\times10^{24}$

Magnetic moment of each dipole,

$$M' = 1.5\times10^{-23}\text{ J T}^{-1}$$

Total dipole moment of sample = $n\times M'$

$= 2\times10^{24}\times1.5\times10^{-23} = 30$ J T^{-1}

As saturation achieved is 15%, therefore, effective dipole moment,

$$M_1 = \frac{15}{100}\times30 = 4.5\text{ J T}^{-1},$$

$B_1 = 0.64$ T, $T_1 = 4.2$ K

$B_2 = 0.98$ T, $T_2 = 2.8$ K

According to Curie's law,

$$\chi_m = \frac{\mu_0 C}{T} = \frac{I}{H} \quad\text{or}\quad I = \frac{\mu_0 CH}{T}$$

As $I \propto M$ and $H \propto B$ $\quad\therefore\quad M \propto \dfrac{B}{T}$,

$$\frac{M_2}{M_1} = \frac{B_2}{B_1}\cdot\frac{T_1}{T_2}$$

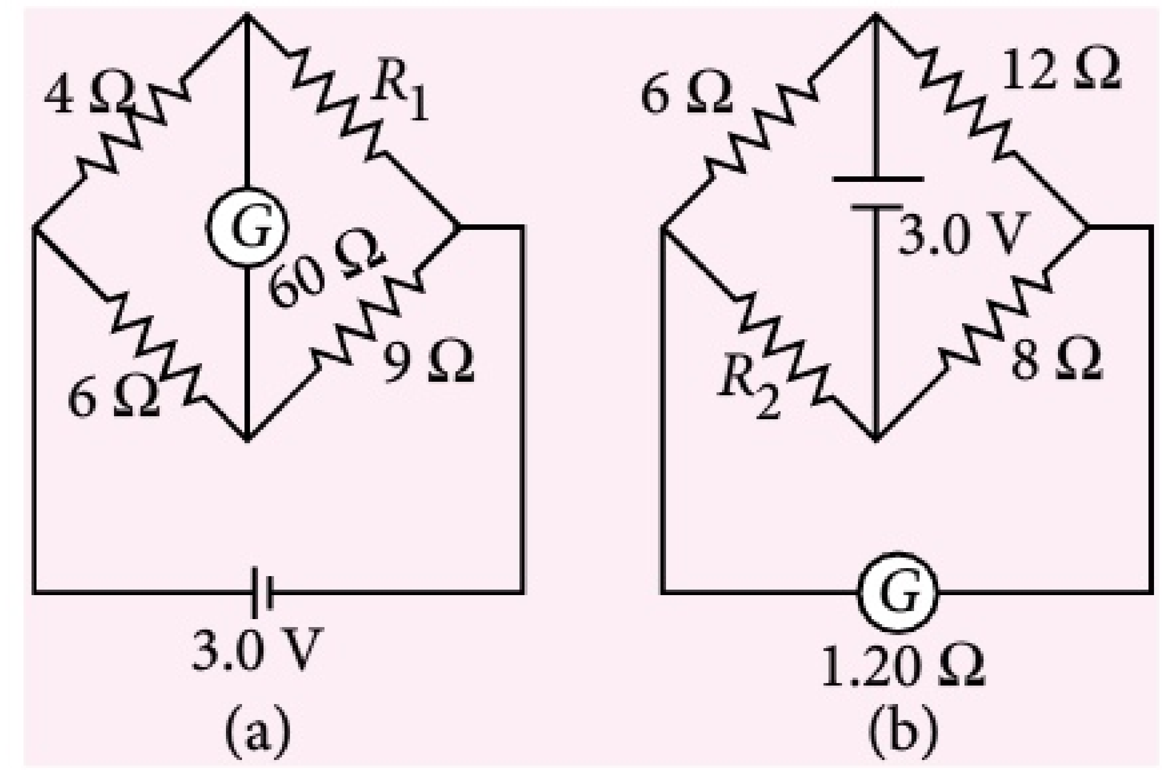
or $\quad M_2 = \dfrac{B_2 T_1 M_1}{T_2 B_1} = \dfrac{0.98\times4.2\times4.5}{2.8\times0.64}$

$M_2 = 10.34$ J T^{-1}

27. Current sensitivity of a galvanometer is the deflection produced in the galvanometer when a unit current flows through it. The galvanometer is said to be more sensitive, if it gives large deflection for small current flowing through it.

S.I. unit of current sensitivity is A^{-1}

or $\dfrac{\text{radian}}{\text{ampere}}$.



Both the arrangements given are of Wheatstone bridge which are balanced. Hence

From figure (a),

$$\frac{4}{R_1} = \frac{6}{9} \quad\text{or}\quad R_1 = 6\ \Omega$$

From figure (b),

$$\frac{6}{12} = \frac{R_2}{8} \quad\text{or}\quad R_2 = 4\ \Omega$$

$$\therefore\ \frac{R_1}{R_2} = \frac{6}{4} = 1.5$$

28. Refer point 3.4(2) page no. 159 (MTG Excel in Physics).

OR

Refer point 3.1(1) page no. 153 and point 3.1(3 (vi)) page no. 154 (MTG Excel in Physics).

29. (a) Refer point 3.4(5, 6) page no. 160 (MTG Excel in Physics).

(b) Refer point 3.3(9) page no. 158 (MTG Excel in Physics).

OR

Refer point 3.3(5) page no. 157 (MTG Excel in Physics).

30. Refer point 3.8(8) page no. 164 (MTG Excel in Physics).

OR

Refer point 3.2(1, 2) page no. 155 (MTG Excel in Physics).

SOLVED PAPER 2014

Maharashtra CET

1. In cyclotron, for a given magnet, radius of the semicircle traced by positive ion is directly proportional to
(v = velocity of positive ion)
(a) v^{-2} (b) v^{-1} (c) v (d) v^2

2. A particle at rest is moved along a straight line by a machine giving constant power. The distance moved by the particle in time t is proportional to
(a) $t^{1/2}$ (b) $t^{2/3}$ (c) t (d) $t^{3/2}$

3. In insulators (C.B. is conduction band and V.B. is valence band)
(a) V.B. is partially filled with electrons.
(b) C.B. is partially filled with electrons.
(c) C.B. is empty and V.B. is filled with electrons.
(d) C.B. is filled with electrons and V.B. is empty.

4. An object of radius R and mass M is rolling horizontally without slipping with speed v. It then rolls up the hill to a maximum height $h = \frac{3v^2}{4g}$. The moment of inertia of the object is
(g = acceleration due to gravity)
(a) $\frac{2}{5}MR^2$ (b) $\frac{MR^2}{2}$
(c) MR^2 (d) $\frac{3}{2}MR^2$

5. In Wheatstone's bridge, three resistors P, Q, R are connected in three arms in order and 4th arm s is formed by two resistors s_1 and s_2 connected in parallel. The condition for bridge to be balanced is $\frac{P}{Q} =$
(a) $\frac{R(s_1+s_2)}{s_1 s_2}$ (b) $\frac{s_1 s_2}{R(s_1+s_2)}$
(c) $\frac{R s_1 s_2}{(s_1+s_2)}$ (d) $\frac{(s_1+s_2)}{R s_1 s_2}$

6. In common base circuit of a transistor, current amplification factor is 0.95. Calculate the emitter current if base current is 0.2 mA.
(a) 2 mA (b) 4 mA (c) 6 mA (d) 8 mA

7. The ratio of magnetic dipole moment of an electron of charge e and mass m in Bohr's orbit in hydrogen atom to its angular momentum is
(a) $\frac{e}{m}$ (b) $\frac{m}{e}$ (c) $\frac{2m}{e}$ (d) $\frac{e}{2m}$

8. Gases exert pressure on the walls of the container because the gas molecules
(a) have finite volume
(b) obey Boyle's law
(c) possess momentum
(d) collide with one another

9. Two coherent sources of intensity ratio α interfere. In interference pattern $\frac{I_{max} - I_{min}}{I_{max} + I_{min}} =$
(a) $\frac{2\alpha}{1+\alpha}$ (b) $\frac{2\sqrt{\alpha}}{1+\alpha}$
(c) $\frac{2\alpha}{1+\sqrt{\alpha}}$ (d) $\frac{1+\alpha}{2\alpha}$

10. Light of wavelength λ_A and λ_B falls on two identical metal plates A and B respectively. The maximum kinetic energy of photoelectrons in K_A and K_B respectively, then which one of the following relations is true? ($\lambda_A = 2\lambda_B$)

(a) $K_A < \frac{K_B}{2}$ (b) $2K_A = K_B$

(c) $K_A = 2K_B$ (d) $K_A > 2K_B$

11. If an electron in hydrogen atom jumps from an orbit of level $n = 3$ to an orbit of level $n = 2$, emitted radiation has a frequency (R = Rydberg's constant, C = velocity of light)

(a) $\frac{3RC}{27}$ (b) $\frac{RC}{25}$

(c) $\frac{8RC}{9}$ (d) $\frac{5RC}{36}$

12. In electromagnetic wave, according to Maxwell, changing electric field gives

(a) stationary magnetic field

(b) conduction current

(c) eddy current

(d) displacement current

13. The de-Broglie wavelength of an electron in 4th orbit is (r = radius of 1st orbit)

(a) $2\pi r$ (b) $4\pi r$ (c) $8\pi r$ (d) $16\pi r$

14. A string of length L and force constant K is stretched to obtain extension l. It is further stretched to obtain extension l_1. The work done in second stretching is

(a) $\frac{1}{2}Kl_1(2l + l_1)$ (b) $\frac{1}{2}Kl_1^2$

(c) $\frac{1}{2}K(l^2 + l_1^2)$ (d) $\frac{1}{2}K(l_1^2 - l^2)$

15. The equiconvex lens has focal length f. If it is cut perpendicular to the principal axis passing through optical centre, then focal length of each half is

(a) $\frac{f}{2}$ (b) f (c) $\frac{3f}{2}$ (d) $2f$

16. If N is the number of turns in a circular coil then the value of self inductance varies as

(a) N^0 (b) N (c) N^2 (d) N^{-2}

17. Surface density of charge on a sphere of radius R in terms of electric intensity E at a distance r in free space is ($\in_0$ = permittivity of free space)

(a) $\in_0 E\left(\frac{R}{r}\right)^2$ (b) $\frac{\in_0 ER}{r^2}$

(c) $\in_0 E\left(\frac{r}{R}\right)^2$ (d) $\frac{\in_0 Er}{R^2}$

18. A body at rest starts sliding from top of a smooth inclined plane and requires 4 second to reach bottom. How much time does it take, starting from rest at top, to cover one-fourth of a distance?

(a) 1 second (b) 2 second

(c) 3 second (d) 4 second

19. In vacuum, to travel distance d, light takes time t and in medium to travel distance $5d$, it takes time T. The critical angle of the medium is

(a) $\sin^{-1}\left(\frac{5T}{t}\right)$ (b) $\sin^{-1}\left(\frac{5t}{3T}\right)$

(c) $\sin^{-1}\left(\frac{5t}{T}\right)$ (d) $\sin^{-1}\left(\frac{3t}{5T}\right)$

20. In electromagnetic spectrum, the frequencies of γ-rays, X-rays and ultraviolet rays are denoted by n_1, n_2 and n_3 respectively then

(a) $n_1 > n_2 > n_3$ (b) $n_1 < n_2 < n_3$

(c) $n_1 > n_2 < n_3$ (d) $n_1 < n_2 > n_3$

21. In LCR series circuit, an alternating e.m.f. e and current i are given by the equations $e = 100 \sin(100t)$ volt,

$i = 100\sin\left(100t + \frac{\pi}{3}\right)$ mA.

The average power dissipated in the circuit will be

(a) 100 W (b) 10 W

(c) 5 W (d) 2.5 W

22. A block resting on the horizontal surface executes S.H.M. in horizontal plane with amplitude A. The frequency of oscillation for which the block just starts to slip is (μ = coefficient of friction, g = gravitational acceleration)

(a) $\frac{1}{2\pi}\sqrt{\frac{\mu g}{A}}$ (b) $\frac{1}{4\pi}\sqrt{\frac{\mu g}{A}}$

(c) $2\pi\sqrt{\frac{A}{\mu g}}$ (d) $4\pi\sqrt{\frac{A}{\mu g}}$

23. A plane sound wave travelling with velocity v in a medium A reaches a point on the interface of medium A and medium B. If velocity of sound in medium B is $2v$, the angle of incidence for total internal reflection of the wave will be greater than (sin 30° = 0.5 and sin 90° = 1)
(a) 15° (b) 30° (c) 45° (d) 90°

24. A gas is compressed isothermally. The r.m.s. velocity of its molecules
(a) increases
(b) decreases
(c) first increases and then decreases
(d) remains the same

25. Two concentric spheres kept in air have radii R and r. They have similar charge and equal surface charge density σ. The electric potential at their common centre is
(ϵ_0 = permittivity of free space)
(a) $\frac{\sigma(R+r)}{\epsilon_0}$ (b) $\frac{\sigma(R-r)}{\epsilon_0}$
(c) $\frac{\sigma(R+r)}{2\epsilon_0}$ (d) $\frac{\sigma(R+r)}{4\epsilon_0}$

26. The velocity of water in river is 9 km/hr of the upper surface. The river is 10 m deep. If the coefficient of viscosity of water is 10^{-2} poise then the shearing stress between horizontal layers of water is
(a) 0.25×10^{-2} N/m^2 (b) 0.25×10^{-3} N/m^2
(c) 0.5×10^{-3} N/m^2 (d) 0.75×10^{-3} N/m^2

27. A sphere P of mass m moving with velocity u collides head-on with another sphere Q of mass m which is at rest. The ratio of final velocity of Q to initial velocity of P is
(e = coefficient of restitution)
(a) $\frac{e-1}{2}$ (b) $\left[\frac{e+1}{2}\right]^{1/2}$
(c) $\frac{e+1}{2}$ (d) $\left[\frac{e+1}{2}\right]^{2}$

28. Magnetic induction produced at the centre of a circular loop carrying current is B. The magnetic moment of the loop of radius R is
(μ_0 = permeability of free space)
(a) $\frac{BR^3}{2\pi\mu_0}$ (b) $\frac{2\pi BR^3}{\mu_0}$
(c) $\frac{BR^2}{2\pi\mu_0}$ (d) $\frac{2\pi BR^2}{\mu_0}$

29. In air, a charged soap bubble of radius r is in equilibrium having outside and inside pressures being equal. The charge on the drop is
(ϵ_0 = permittivity of free space, T = surface tension of soap solution)
(a) $4\pi r^2\sqrt{\frac{2T\epsilon_0}{r}}$ (b) $4\pi r^2\sqrt{\frac{4T\epsilon_0}{r}}$
(c) $4\pi r^2\sqrt{\frac{6T\epsilon_0}{r}}$ (d) $4\pi r^2\sqrt{\frac{8T\epsilon_0}{r}}$

30. A block is pushed momentarily on a horizontal surface with initial velocity v. If μ is the coefficient of sliding friction between the block and surface, the block will come to rest after time (g = acceleration due to gravity)
(a) $\frac{v}{\mu g}$ (b) $\frac{vg}{\mu}$ (c) $\frac{v\mu}{g}$ (d) $\frac{\mu g}{v}$

31. Two charges of equal magnitude q are placed in air at a distance $2a$ apart and third charge $-2q$ is placed at midpoint. The potential energy of the system is
(ϵ_0 = permittivity of free sapce)
(a) $-\frac{q^2}{8\pi\epsilon_0 a}$ (b) $-\frac{3q^2}{8\pi\epsilon_0 a}$
(c) $-\frac{5q^2}{8\pi\epsilon_0 a}$ (d) $-\frac{7q^2}{8\pi\epsilon_0 a}$

32. An electron in potentiometer wire experiences a force 2.4×10^{-19} N. The length of potentiometer wire is 6 m. The e.m.f. of the battery connected across the wire is
(electronic charge = 1.6×10^{-19} C)
(a) 6 V (b) 9 V
(c) 12 V (d) 15 V

33. The dimensional formula for Reynold's number is
(a) $[L^0 M^0 T^0]$ (b) $[L^1 M^1 T^1]$
(c) $[L^{-1} M^1 T^1]$ (d) $[L^1 M^1 T^{-1}]$

34. Calculate angular velocity of earth so that acceleration due to gravity at 60° latitude becomes zero.
(Radius of earth = 6400 km, gravitational acceleration at poles = 10 m/s^2, cos 60° = 0.5)
(a) 7.8×10^{-2} rad/s (b) 0.5×10^{-3} rad/s
(c) 1×10^{-3} rad/s (d) 2.5×10^{-3} rad/s

35. A stationary object explodes into masses m_1 and m_2. They move in opposite directions with velocities V_1 and V_2. The ratio of kinetic energy E_1 to kinetic energy E_2 is
(a) $\frac{m_2}{m_1}$ (b) $\frac{m_1}{m_2}$ (c) $\frac{2m_2}{m_1}$ (d) $\frac{2m_1}{m_2}$

36. The moment of inertia of a thin uniform rod rotating about the perpendicular axis passing through one end is I. The same rod is bent into a ring and its moment of inertia about the diameter is I_1. The ratio $\frac{I}{I_1}$ is
(a) $\frac{4\pi}{3}$ (b) $\frac{8\pi^2}{3}$ (c) $\frac{5\pi}{3}$ (d) $\frac{8\pi^2}{5}$

37. Three identical spheres each of mass 1 kg are placed touching one another with their centres in a straight line. Their centres are marked as A, B, C respectively. The distance of centre of mass of the system from A is
(a) $\frac{AB+AC}{2}$ (b) $\frac{AB+BC}{2}$
(c) $\frac{AC-AB}{3}$ (d) $\frac{AB+AC}{3}$

38. The relation between force F and density d is $F=\frac{x}{\sqrt{d}}$. The dimensions of x are
(a) $[L^{-1/2} M^{3/2} T^{-2}]$ (b) $[L^{-1/2} M^{1/2} T^{-2}]$
(c) $[L^{-1} M^{3/2} T^{-2}]$ (d) $[L^{-1} M^{1/2} T^{-2}]$

39. When a wave travels in a medium, displacement of a particle is given by $y = a\sin 2\pi(bt - cx)$ where a, b, c are constants. The maximum particle velocity will be twice the wave velocity if
(a) $b = ac$ (b) $b=\frac{1}{ac}$
(c) $c = \pi a$ (d) $c=\frac{1}{\pi a}$

40. Electromagnets are made of soft iron because soft iron has
(a) high susceptibility and low retentivity
(b) low susceptibility and high retentivity
(c) low susceptibility and low retentivity
(d) high susceptibility and high retentivity

41. The masses of three copper wires are in the ratio 1 : 3 : 5 and their lengths are in the ratio 5 : 3 : 1. The ratio of their resistance is
(a) 25 : 1 : 125 (b) 1 : 125 : 25
(c) 125 : 1 : 25 (d) 125 : 25 : 1

42. A body of mass m is raised to a height $10R$ from the surface of earth, where R is the radius of earth. The increase in potential energy is (G = universal constant of gravitation, M = mass of earth and g = acceleration due to gravity)
(a) $\frac{GMm}{11R}$ (b) $\frac{GMm}{10R}$
(c) $\frac{mgR}{11G}$ (d) $\frac{10\,GMm}{11R}$

43. The angle θ between the vector $\vec{p}=\hat{i}+\hat{j}+\hat{k}$ and unit vector along x-axis is
(a) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (b) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$
(c) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (d) $\cos^{-1}\left(\frac{1}{2}\right)$

44. A small metal ball of mass m is dropped in a liquid contained in a vessel, attains a terminal velocity v. If a metal ball of same material but of mass $8m$ is dropped in same liquid then the terminal velocity will be
(a) v (b) $2v$ (c) $4v$ (d) $8v$

45. A wooden block of mass 8 kg slides down an inclined plane of inclination 30° to the horizontal with constant acceleration 0.4 m/s^2. The force of friction between the block and inclined plane is (g = 10 m/s^2)
(a) 12.2 N (b) 24.4 N
(c) 36.8 N (d) 48.8 N

SOLUTIONS

1. **(c):** Radius of the semicircle traced by positive ion is given by

$$r = \frac{mv}{qB}$$

where m and q be mass and charge of positive ion. So, for a given magnet,

$$r \propto v$$

2. **(d) :** Power, $P = Fv = (ma)v = m\frac{dv}{dt}v$

$$vdv = \frac{P}{m}dt$$

Integrating both sides,

$$\int_0^v vdv = \frac{P}{m}\int_0^t dt \qquad (\because P \text{ is a constant})$$

$$\frac{v^2}{2} = \frac{P}{m}t \quad \text{or} \quad v = \sqrt{\frac{2P}{m}}t^{1/2}$$

As $v = \frac{dx}{dt}$

$$\therefore \; dx = vdt = \sqrt{\frac{2P}{m}}t^{1/2}dt$$

Integrating both sides,

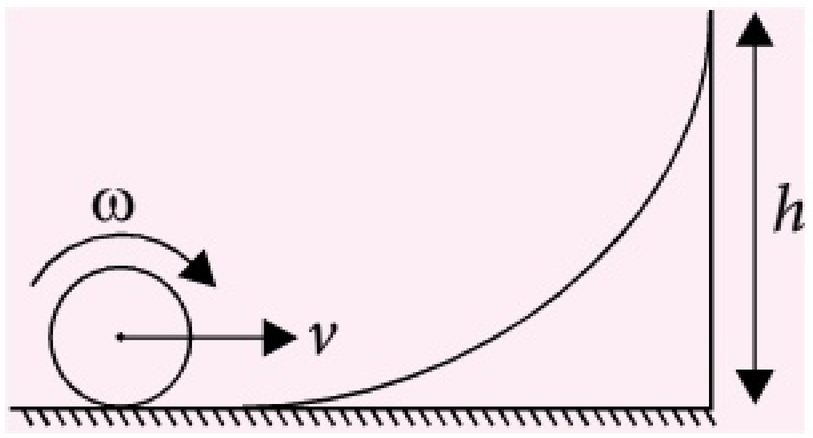
$$\int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{1/2}dt$$

$$x = \sqrt{\frac{2P}{m}}\frac{t^{3/2}}{(3/2)} = \frac{2}{3}\sqrt{\frac{2P}{m}}t^{3/2} = \sqrt{\frac{8P}{9m}}t^{3/2}$$

$$\therefore \; x \propto t^{3/2}$$

3. **(c):** In insulators conduction band is empty and valence band is completely filled with electrons.

4. **(b) :** The kinetic energy of the rolling object is converted into potential energy at height $h\left(=\frac{3v^2}{4g}\right)$.



According to the law of conservation of mechanical energy, we get

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 = Mgh$$

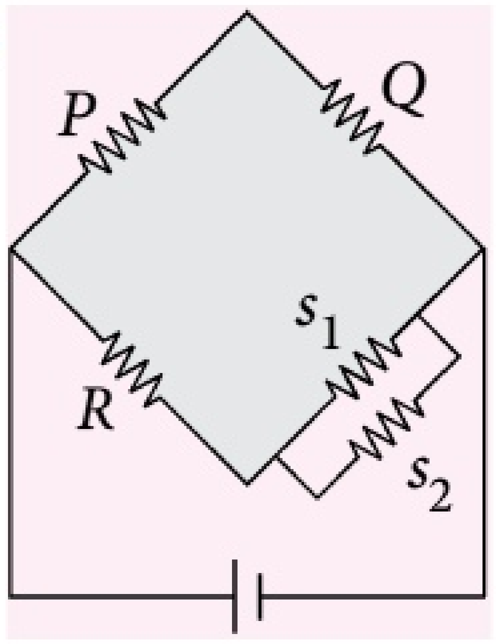
$$\frac{1}{2}Mv^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2 = Mg\left(\frac{3v^2}{4g}\right) \qquad \left(\because \; \omega = \frac{v}{R}\right)$$

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\frac{v^2}{R^2} = \frac{3}{4}Mv^2$$

$$\frac{1}{2}I\frac{v^2}{R^2} = \frac{3}{4}Mv^2 - \frac{1}{2}Mv^2 = \frac{1}{4}Mv^2$$

$$I = \frac{1}{2}MR^2$$

5. **(a):**



As s_1 and s_2 are in parallel, so their equivalent resistance is

$$\frac{1}{s} = \frac{1}{s_1} + \frac{1}{s_2} = \frac{s_2 + s_1}{s_1 s_2}$$

$$s = \frac{s_1 s_2}{s_1 + s_2}$$

For bridge to be balanced, $\frac{P}{Q} = \frac{R}{s}$

$$\therefore \quad \frac{P}{Q} = \frac{R}{s_1 s_2/(s_1 + s_2)} = \frac{R(s_1 + s_2)}{s_1 s_2}$$

6. **(b) :** Here, $\alpha = 0.95$, $I_B = 0.2$ mA

As $\alpha = \frac{I_C}{I_E} = 0.95$

$\therefore \quad I_C = 0.95\,I_E$

As $\quad I_E = I_B + I_C$

$\therefore \quad I_E = I_B + 0.95\,I_E$

$0.05\,I_E = I_B$

$$I_E = \frac{I_B}{0.05} = \frac{0.2\text{ mA}}{0.05} = 4\text{ mA}$$

7. (d) : $\frac{\text{Magnetic dipole moment of an electron}}{\text{Angular momentum}} = \frac{e}{2m}$

This ratio is called gyromagnetic ratio.

8. (c): Gases exert pressure on the walls of the container because the gas molecules possess momentum.

9. (b) : Let two coherent sources of intensity I_1 and I_2 interfere. Then

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} = \frac{\left(\sqrt{\frac{I_1}{I_2}}+1\right)^2}{\left(\sqrt{\frac{I_1}{I_2}}-1\right)^2}$$

As $\frac{I_1}{I_2} = \alpha$ (Given)

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{\alpha}+1\right)^2}{\left(\sqrt{\alpha}-1\right)^2} = \left(\frac{\sqrt{\alpha}+1}{\sqrt{\alpha}-1}\right)^2$$

Then,

$$\frac{I_{max}-I_{min}}{I_{max}+I_{min}} = \frac{\frac{I_{max}}{I_{min}}-1}{\frac{I_{max}}{I_{min}}+1}$$

$$= \frac{\left(\frac{\sqrt{\alpha}+1}{\sqrt{\alpha}-1}\right)^2-1}{\left(\frac{\sqrt{\alpha}+1}{\sqrt{\alpha}-1}\right)^2+1}$$

$$= \frac{\left(\sqrt{\alpha}+1\right)^2-\left(\sqrt{\alpha}-1\right)^2}{\left(\sqrt{\alpha}+1\right)^2+\left(\sqrt{\alpha}-1\right)^2}$$

$$= \frac{\alpha+1+2\sqrt{\alpha}-\alpha-1+2\sqrt{\alpha}}{\alpha+1+2\sqrt{\alpha}+\alpha+1-2\sqrt{\alpha}}$$

$$= \frac{4\sqrt{\alpha}}{2+2\alpha} = \frac{4\sqrt{\alpha}}{2(1+\alpha)} = \frac{2\sqrt{\alpha}}{1+\alpha}$$

10. (a): According to Einstein's photoelectric equation

$$K_A = \frac{hc}{\lambda_A} - \phi_0 \quad \text{...(i)}$$

and $K_B = \frac{hc}{\lambda_B} - \phi_0$ ...(ii)

As $\lambda_A = 2\lambda_B$ (Given)

$$\therefore \quad K_A = \frac{hc}{2\lambda_B} - \phi_0$$

$$= \frac{hc}{2\lambda_B} - \frac{\phi_0}{2} - \frac{\phi_0}{2}$$

$$= \frac{1}{2}\left(\frac{hc}{\lambda_B} - \phi_0\right) - \frac{\phi_0}{2}$$

$$= \frac{K_B}{2} - \frac{\phi_0}{2} \quad \text{(Using (ii))}$$

Thus, $K_A < \frac{K_B}{2}$

11. (d) : When an electron jumps from n = 3 to n = 2, the frequency of emitted radiation is

$$\upsilon = RC\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = RC\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$= RC\left[\frac{9-4}{36}\right] = \frac{5RC}{36}$$

12. (d) : According to Maxwell, changing electric field gives displacement current. Displacement current exists due to time-varying electric field and is given by

$$I_d = \epsilon_0 \frac{d\phi_E}{dt}$$

13. (c): Radius of n^{th} orbit is

$r_n = n^2 r$ where r is the radius of 1^{st} orbit.

If λ is the de-Broglie wavelength of an electron while revolving in n^{th} orbit of radius r_n, then

$2\pi r_n = n\lambda$

$$\lambda = \frac{2\pi r_n}{n}$$

For 4^{th} orbit, n = 4

$$\therefore \quad \lambda = \frac{2\pi r_4}{4} = \frac{2\pi(4)^2 r}{4} = 8\pi r$$

14. (a): Potential energy of the string after the first stretching is

$$U_1 = \frac{1}{2}Kl^2$$

Potential energy of the string after the second stretching is

$$U_2 = \frac{1}{2}K(l + l_1)^2$$

Work done in second stretching is

$$W = U_2 - U_1 = \frac{1}{2}K(l + l_1)^2 - \frac{1}{2}Kl^2$$

$$= \frac{1}{2}K((l + l_1)^2 - l^2)$$

$$= \frac{1}{2}K(l^2 + l_1^2 + 2ll_1 - l^2)$$

$$= \frac{1}{2}K(l_1^2 + 2ll_1) = \frac{1}{2}Kl_1(l_1 + 2l)$$

15. (d) : According to lens maker's formula

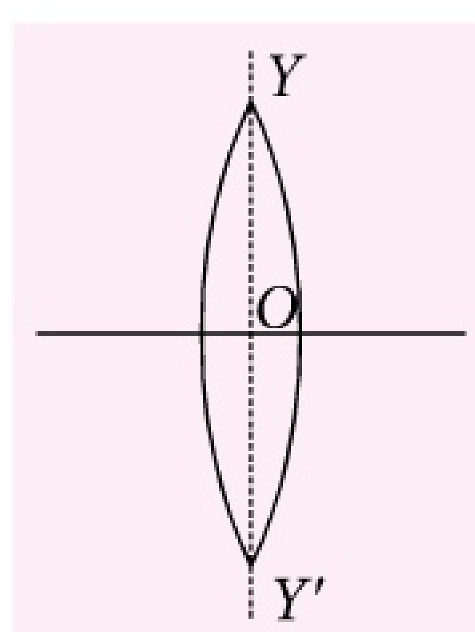
$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For the given lens,

$R_1 = R$, $R_2 = -R$

$$\therefore \quad \frac{1}{f} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) = \frac{2(\mu - 1)}{R}$$

$$f = \frac{R}{2(\mu - 1)} \qquad \text{...(i)}$$



When the lens is cut into two halves along YOY', then each half behaves as a plano convex lens.

For each half lens,

$R_1 = R$, $R_2 = \infty$

If f' is the focal length of each half, then

$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right) = \frac{(\mu - 1)}{R}$$

$$f' = \frac{R}{(\mu - 1)} \qquad \text{...(ii)}$$

Divide eqn. (ii) by eqn. (i), we get

$$\frac{f'}{f} = 2 \quad \text{or} \quad f' = 2f$$

16. (c) : Self inductance of a circular coil is

$$L = \frac{1}{2}\mu_0 N^2 \pi R$$

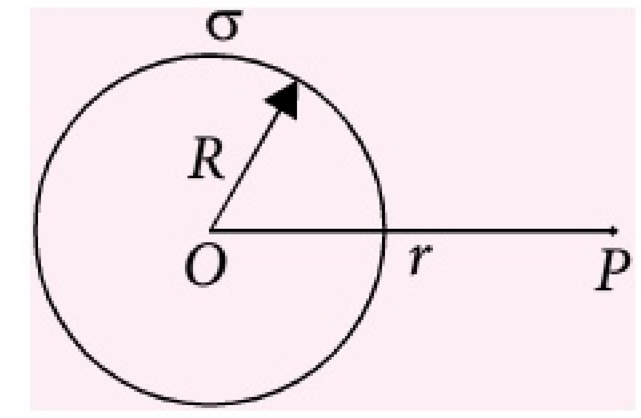
where,

N = Number of turns in the coil

R = Radius of the coil

$\therefore \quad L \propto N^2$

17. (c) : Electric field intensity at a distance r from the centre of sphere is



$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} \quad (\text{For } r > R)$$

Surface charge density of the sphere is

$$\sigma = \frac{Q}{4\pi R^2}$$

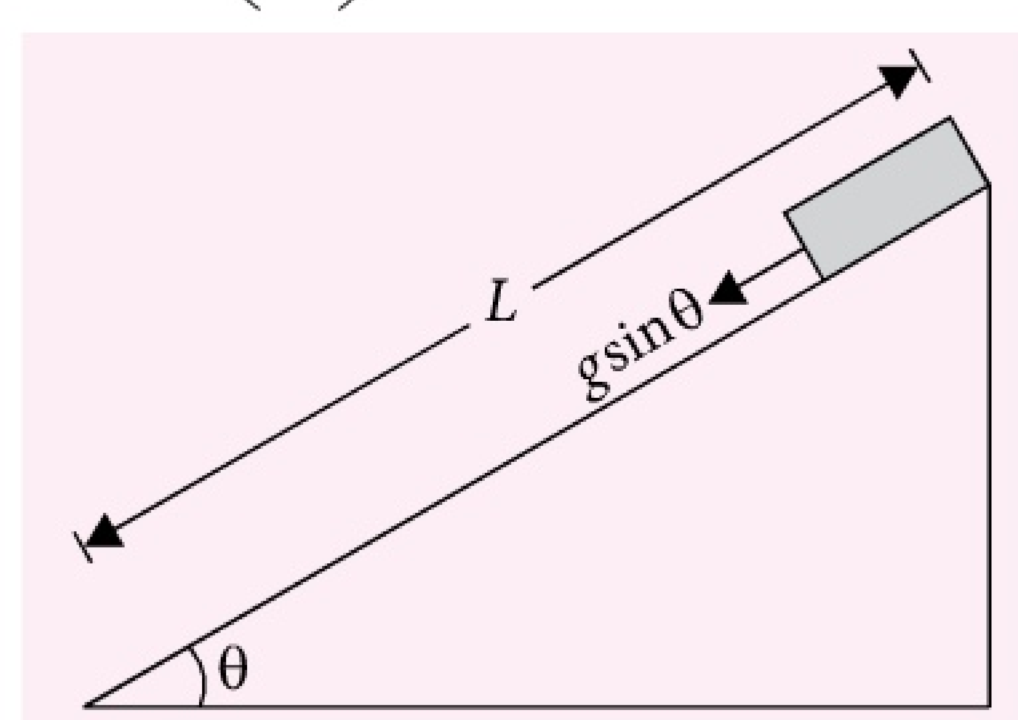
$$Q = \sigma 4\pi R^2$$

$$\therefore \quad E = \frac{1}{4\pi\epsilon_0}\frac{(\sigma 4\pi R^2)}{r^2}$$

$$= \frac{\sigma R^2}{\epsilon_0 r^2}$$

$$\text{or} \quad \sigma = \epsilon_0 E\left(\frac{r}{R}\right)^2$$

18. (b) :



Let L be length of the inclined plane inclined at an angle θ.

Acceleration of the body down the plane is

$a = g\sin\theta$

As the body starts from rest, therefore its initial velocity $u = 0$.

Then,

$$L = \frac{1}{2}g\sin\theta(4)^2$$

$$\text{and} \quad \frac{L}{4} = \frac{1}{2}g\sin\theta(t)^2$$

$$\therefore \quad \frac{L/4}{L} = \frac{t^2}{4^2}$$

$$t^2 = 4$$

or $\quad t = 2$ second

19. (c) : In vacuum,

$d = ct$

where c is the velocity of light in vacuum.

or $c = \frac{d}{t}$ ...(i)

In medium,

$5d = vT$

where v is the velocity of light in medium.

$v = \frac{5d}{T}$ ... (ii)

Refractive index of the medium is

$$\mu = \frac{\text{Velocity of light in vacuum}}{\text{Velocity of light in medium}} = \frac{c}{v}$$

Let i_c be critical angle of the medium. Then

$$\sin i_c = \frac{1}{\mu} = \frac{v}{c} = \left(\frac{5d/T}{d/t}\right) = \frac{5t}{T} \text{(Using (i) and (ii))}$$

or $i_c = \sin^{-1}\left(\frac{5t}{T}\right)$

20. (a) : The correct order of frequencies of given electromagnetic waves is

$n_1 > n_2 > n_3$

21. (d) : Here,

$e = 100 \sin(100t)$ volt

$i = 100 \sin\left(100t + \frac{\pi}{3}\right)$ mA

$\therefore$ $e_0 = 100$ volt

$i_0 = 100$ mA $= 100 \times 10^{-3}$ A

$\phi = \frac{\pi}{3}$

Average power dissipated in the circuit is

$$P_{av} = \frac{1}{2} e_0 i_0 \cos\phi$$

$$= \frac{1}{2} \times 100 \times 100 \times 10^{-3} \times \cos\frac{\pi}{3}$$

$$= \frac{1}{2} \times 10 \times \frac{1}{2} = \frac{5}{2} \text{ W} = 2.5 \text{ W}$$

22. (a): Let m be mass of the block.

When the block is about to slip, then

Force of friction = Centrifugal force

$$\mu mg = m\omega^2 A$$

$$\omega^2 = \frac{\mu g}{A} \quad \text{or} \quad \omega = \sqrt{\frac{\mu g}{A}}$$

As $\omega = 2\pi\upsilon$

$$\therefore \quad \upsilon = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\mu g}{A}}$$

23. (b) : Let i be angle of incidence and i_c be the critical angle.

For total internal reflection,

$i > i_c$

Here, $\sin i_c = \frac{v}{2v} = \frac{1}{2}$

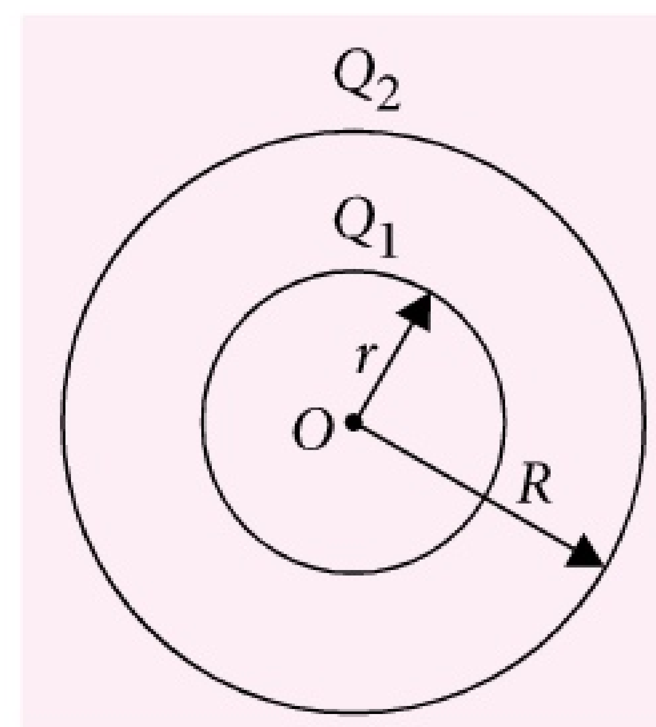
or $i_c = \sin^{-1}\left(\frac{1}{2}\right) = \sin^{-1}(0.5) = 30°$

$\therefore$ $i > 30°$

24. (d) : As the gas is compressed isothermally, therefore its temperature remains the same.

Since $v_{rms} \propto \sqrt{T}$, hence the r.m.s. velocity of its molecules remains the same.

25. (a):



As both the spheres have equal surface charge density σ, therefore charges on the spheres are

$Q_1 = \sigma 4\pi r^2$ and $Q_2 = \sigma 4\pi R^2$

Electric potential at their common centre O is

$$V = \frac{1}{4\pi\epsilon_0}\frac{Q_1}{r} + \frac{1}{4\pi\epsilon_0}\frac{Q_2}{R}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{\sigma 4\pi r^2}{r} + \frac{\sigma 4\pi R^2}{R}\right]$$

$$= \frac{4\pi\sigma}{4\pi\epsilon_0}[r+R] = \frac{\sigma(R+r)}{\epsilon_0}$$

26. (b) : Here,

Coefficient of viscosity of water, $\eta = 10^{-2}$ poise $= 10^{-3}$ Pa s

As the velocity at the bottom of the river is zero, therefore change in velocity of river from surface to bottom is

$$dv = 9 \text{ km/hr} = 9 \times \frac{5}{18} \text{ m/s} = 2.5 \text{ m/s}$$

Change of depth from surface to bed of the river,

$dx = 10$ m

$\therefore$ Velocity gradient $= \dfrac{dv}{dx}$

$$= \frac{2.5 \text{ m/s}}{10 \text{ m}} = 0.25 \text{ s}^{-1}$$

As viscous force, $F = \eta A \dfrac{dv}{dx}$

$\therefore$ Shearing stress $= \dfrac{F}{A} = \eta \dfrac{dv}{dx}$

$$= (10^{-3} \text{ Pa s})(0.25 \text{ s}^{-1})$$
$$= 0.25 \times 10^{-3} \text{ Pa}$$
$$= 0.25 \times 10^{-3} \text{ N/m}^2$$

27. (c): Let v_P and v_Q be the final velocities of the spheres P and Q after collision.

According to law of conservation of momentum,

$mu = mv_P + mv_Q$

or $\quad v_P + v_Q = u \qquad$ ...(i)

Coefficient of restitution, $e = \dfrac{v_Q - v_P}{u}$

or $\quad v_Q - v_P = ue \qquad$ ...(ii)

Adding eqns. (i) and (ii), we get

$2v_Q = u + ue = u(1 + e)$

$$v_Q = \frac{u(1+e)}{2}$$

The required ratio is

$$\frac{v_Q}{u} = \frac{1+e}{2}$$

28. (b) : Magnetic induction at the centre of a circular loop of radius R carrying current I is

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I}{R} = \frac{\mu_0 I}{2R}$$

$$\therefore \quad I = \frac{2RB}{\mu_0}$$

Area of the loop, $A = \pi R^2$

Magnetic moment of the loop,

$$M = IA = \left(\frac{2RB}{\mu_0}\right)(\pi R^2) = \frac{2\pi BR^3}{\mu_0}$$

29. (d) : Excess pressure inside the soap bubble is

$$P = \frac{4T}{r}$$

As the pressure inside and outside is the same, so pressure of the charged bubble is

$$P_{\text{electro}} = \frac{4T}{r}$$

$$\frac{\sigma^2}{2\epsilon_0} = \frac{4T}{r}$$

where σ is the surface charge density of the bubble

$$\sigma = \sqrt{\frac{8T\epsilon_0}{r}}$$

$\therefore$ Charge on the bubble is

$$Q = 4\pi r^2 \sigma = 4\pi r^2 \sqrt{\frac{8T\epsilon_0}{r}}$$

30. (a)

31. (d) :

The potential energy of the system is

$$U = \frac{1}{4\pi\epsilon_0}\frac{(q)(-2q)}{a} + \frac{1}{4\pi\epsilon_0}\frac{(q)(q)}{2a} + \frac{1}{4\pi\epsilon_0}\frac{(q)(-2q)}{a}$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{-2q^2}{a} + \frac{q^2}{2a} + \frac{-2q^2}{a}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{-4q^2 + q^2 - 4q^2}{2a}\right]$$

$$= -\frac{7q^2}{8\pi\epsilon_0 a}$$

32. (b) : Force on an electron, $F = eE$

$\therefore$ Electric field,

$$E = \frac{F}{e} = \frac{2.4\times10^{-19} \text{ N}}{1.6\times10^{-19} \text{ C}} = \frac{3}{2} \text{ N C}^{-1}$$

Potential difference across the wire is

$$V = El = \frac{3}{2} \text{ N C}^{-1} \times 6 \text{ m} = 9 \text{ V}$$

$\therefore$ Emf of the battery $= V = 9$ V.

33. (a): Reynold's number is a dimensionless quantity. Therefore, its dimensional formula is $[\text{L}^0 \text{M}^0 \text{T}^0]$.

34. (d): Acceleration due to gravity at latitude λ is

$g_\lambda = g - R\omega^2\cos^2\lambda$

where R is the radius and ω is the angular velocity of earth.

At poles, $\lambda = 90°$

$\therefore \quad g_p = g - R\omega^2\cos^2 90° = g \; (\because \cos 90° = 0) \quad$ ...(i)

At latitude $\lambda = 60°$

$g_{60°} = g - R\omega^2\cos^2 60°$

$$= g_p - \frac{1}{4}R\omega^2 \qquad \text{(Using (i))}$$

As per question, $g_{60°}$ = zero

$$\therefore \quad \frac{1}{4}R\omega^2 = g_p$$

$$\omega = \sqrt{\frac{4g_p}{R}} = \sqrt{\frac{4\times 10\ \text{m/s}^2}{6400\times 10^3\ \text{m}}} = \frac{1}{4}\times 10^{-2}\ \text{rad/s}$$

$= 2.5 \times 10^{-3}$ rad/s

35. (a): According to law of conservation of momentum,

$0 = m_1V_1 + m_2V_2$

$$V_2 = -\frac{m_1}{m_2}V_1$$

–ve sign shows that both are moving in opposite directions.

In terms of magnitude

$$V_2 = \frac{m_1}{m_2}V_1 \quad \text{or} \quad \frac{V_2}{V_1} = \frac{m_1}{m_2} \qquad \text{...(i)}$$

Kinetic energy of mass m_1, $E_1 = \frac{1}{2}m_1V_1^2$

Kinetic energy of mass m_2, $E_2 = \frac{1}{2}m_2V_2^2$

The required ratio is

$$\frac{E_1}{E_2} = \frac{\frac{1}{2}m_1V_1^2}{\frac{1}{2}m_2V_2^2} = \frac{m_1}{m_2}\frac{V_1^2}{V_2^2}$$

$$= \frac{m_1}{m_2}\left(\frac{m_2}{m_1}\right)^2 = \frac{m_2}{m_1} \qquad \text{(Using (i))}$$

36. (b) : Let M be mass and L be length of the rod. Moment of inertia of the rod about the perpendicular axis passing through one end is

$$I = \frac{1}{3}ML^2$$

When it is bent into a ring of radius R, then

$2\pi R = L$

$$R = \frac{L}{2\pi} \qquad \text{...(i)}$$

Moment of inertia of the ring about the diameter is

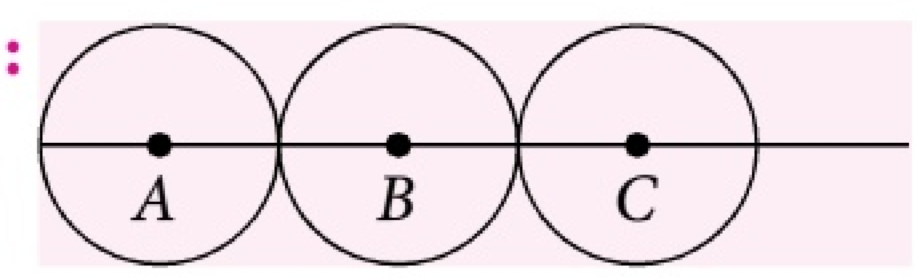
$$I_1 = \frac{1}{2}MR^2 = \frac{1}{2}M\left(\frac{L}{2\pi}\right)^2 \qquad \text{(Using (i))}$$

$$= \frac{1}{8\pi^2}ML^2$$

The required ratio is

$$\frac{I}{I_1} = \frac{\frac{1}{3}ML^2}{\frac{1}{8\pi^2}ML^2} = \frac{8\pi^2}{3}$$

37. (d) :



As it is clear from the symmetry of the figure, that centre of mass of the system is at B.

$\therefore$ Its distance from A is

$$x = \frac{m_A\times 0 + m_B\times AB + m_C\times AC}{m_A + m_B + m_C}$$

Here, $m_A = m_B = m_C = 1$ kg

$$\therefore \quad x = \frac{AB + AC}{3}$$

38. (a): Given : $F = \dfrac{x}{\sqrt{d}}$

$\therefore \quad x = F\sqrt{d}$

$[x] = [L^1\,M^1\,T^{-2}][L^{-3}\,M^1\,T^0]^{1/2}$

$= [L^1\,M^1\,T^{-2}][L^{-3/2}\,M^{1/2}\,T^0]$

$= [L^{-1/2}\,M^{3/2}\,T^{-2}]$

39. (d) : The given equation is

$y = a\sin 2\pi(bt - cx)$

Comparing the given equation with the standard wave equation

$$y = a\sin 2\pi\left(\upsilon t - \frac{x}{\lambda}\right)$$

We get,

Frequency of the wave, $\upsilon = b$

Wavelength of the wave, $\lambda = \dfrac{1}{c}$

$\therefore$ Velocity of the wave, $v = \upsilon\lambda = \dfrac{b}{c}$

Particle velocity,

$$v_{\text{Pa}} = \frac{dy}{dt} = a2\pi b\cos 2\pi(bt - cx)$$

$(v_{Pa})_{max} = a2\pi b$

As per question, $(v_{Pa})_{max} = 2v$

$$\therefore \quad a2\pi b = \frac{2b}{c} \quad \text{or} \quad c = \frac{1}{\pi a}$$

40. (a): Electromagnets are made of soft iron because soft iron has high susceptibility and low retentivity.

41. (*): Mass, M = volume × density = $Al \times d$

$$\therefore \quad A = \frac{M}{ld}$$

Resistance, $R = \frac{\rho l}{A} = \frac{\rho l}{(M/ld)} = \frac{\rho l^2 d}{M}$

Since ρ and d are same for all the three wires,

$$\therefore \quad R \propto \frac{l^2}{M}$$

Thus,

$$R_1 : R_2 : R_3 = \frac{l_1^2}{M_1} : \frac{l_2^2}{M_2} : \frac{l_3^2}{M_3}$$

$$= \frac{5^2}{1} : \frac{3^2}{3} : \frac{1^2}{5} = 125 : 15 : 1$$

* None of the given options is correct.

42. (d) : Potential energy of the body of mass m on the surface of the earth is

$$U_i = -\frac{GMm}{R}$$

Potential energy of the same body at a height $h(= 10R)$ from the surface of earth is

$$U_f = -\frac{GMm}{R+h} = -\frac{GMm}{R+10R} = -\frac{GMm}{11R}$$

$\therefore$ Increase in potential energy,

$$\Delta U = U_f - U_i = -\frac{GMm}{11R} - \left(-\frac{GMm}{R}\right)$$

$$= -\frac{GMm}{11R} + \frac{GMm}{R}$$

$$= \frac{GMm}{R}\left[1 - \frac{1}{11}\right] = \frac{10GMm}{11R}$$

43. (a): Here, $\vec{p} = \hat{i} + \hat{j} + \hat{k}$

Unit vector along x-axis $= \hat{i}$

$$\therefore \quad \cos\theta = \frac{\vec{p}\cdot\hat{i}}{|\vec{p}||\hat{i}|}$$

$$\cos\theta = \frac{(\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i})}{\sqrt{(1)^2+(1)^2+(1)^2}\sqrt{(1)^2}} = \frac{1}{\sqrt{3}}$$

$$\text{or} \quad \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

44. (c): Let r and r' be radii of the ball of mass m and $8m$ respectively. Then

$$m = \frac{4}{3}\pi r^3 \rho \quad \text{and} \quad 8m = \frac{4}{3}\pi r'^3 \rho$$

$$\therefore \quad \frac{m}{8m} = \frac{r^3}{r'^3} \quad \text{or} \quad \frac{1}{8} = \frac{r^3}{r'^3} \quad \text{or} \quad r' = 2r$$

Terminal velocity, $v = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

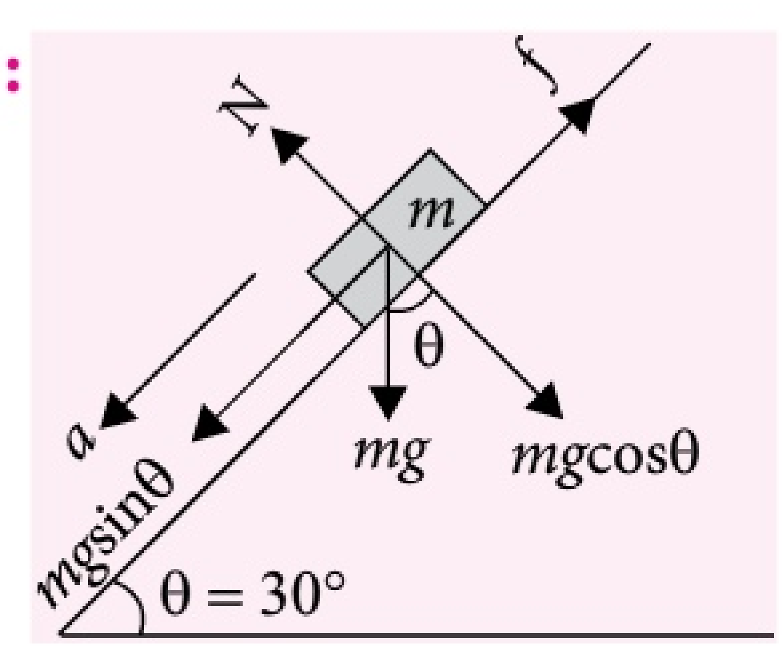
Since $(\rho - \sigma)$ and η are same for both balls

$\therefore \quad v \propto r^2$

$$\text{Thus,} \quad \frac{v'}{v} = \left(\frac{r'}{r}\right)^2 = \left(\frac{2r}{r}\right)^2 = 4$$

or $\quad v' = 4v$

45. (c):



Here, mass of the block, $m = 8$ kg

Acceleration of the block, $a = 0.4$ m/s^2

Angle of inclination, $\theta = 30°$

Let f be force of friction between the block and inclined plane.

$\therefore$ Net downward force on the block is

$$F = mg\sin\theta - f$$
$$ma = mg\sin\theta - f$$
$$f = mg\sin\theta - ma$$
$$= m(g\sin\theta - a)$$
$$= 8(10\sin 30° - 0.4)$$
$$= 8(5 - 0.4)$$
$$= 8 \times 4.6 \text{ N} = 36.8 \text{ N}$$

BRAIN MAP

MOTION IN A PLANE

Galileo proposed what we now call the law of compound motion, according to which the motion in one dimension has no effect on motion in another dimension

Scalars and Vectors

- **Scalar quantities :** The physical quantities which have only magnitude and no direction are called scalar quantities.
- **Vector quantities :** The physical quantities which have both magnitude and direction and obey the laws of vector addition are called vector quantities.

Types of Vectors

- **Polar vectors :** The vectors which have a starting point or a point of application are called polar vectors.
- **Axial vectors :** The vectors which represent rotational effect and act along the axis of rotation in accordance with right hand screw rule are called axial vectors.

Equal, Negative and Zero (or Null) Vectors

- **Equal vectors :** Two vectors are said to be equal if they have same magnitude and direction regardless of their initial positions.
- **Negative vector :** It is a vector having same magnitude but direction opposite to that of a given vector.
- **Null vector :** It is a vector whose magnitude is zero but its direction is not defined.
- **Properties of a null vector:**

(i) $\vec{A}+\vec{0}=\vec{A}$ (ii) $\lambda\vec{0}=\vec{0}$ where λ is a scalar. (iii) $0\vec{A}=\vec{0}$

Projectile Motion

- Any body projected into space, such that it moves under the effect of gravity alone is called a projectile. The path followed by a projectile is called its trajectory which is always a parabola.
- A projectile executes two independent motion simultaneously:
  (i) uniform horizontal motion and
  (ii) uniform accelerated downward motion.

Parallelogram Law of Vectors

- It states that if two vectors acting simultaneously at a point can be represented both in magnitude and direction by the two adjacent sides of a parallelogram, then the resultant is represented completely (both in magnitude and direction) by the diagonal of the parallelogram passing through that point.

$$R=\sqrt{P^2+Q^2+2PQ\cos\theta}\,;\ \tan\alpha=\frac{Q\sin\theta}{P+Q\cos\theta}$$

where $\vec{P}$ and $\vec{Q}$ are two vectors inclined to each other at an angle θ and R is their resultant. The resultant $\vec{R}$ makes an angle α with $\vec{P}$.

Projectile Fired Horizontally

- Suppose a body is projected horizontally with velocity u from a height h above the ground. Let it reach the point (x, y) after time t. Then

(i) Position of the projectile after time t: $x=ut,\ y=\frac{1}{2}gt^2$

(ii) Equation of trajectory : $y=\frac{g}{2u^2}x^2$

(iii) Velocity after time t: $v=\sqrt{u^2+g^2t^2}$; $\beta=\tan^{-1}\frac{gt}{u}$

(iv) Time of flight, $T=\sqrt{\frac{2h}{g}}$

(v) Horizontal range, $R=u\times T=u\sqrt{\frac{2h}{g}}$.

Rectangular Resolution of a Vector in a Plane

- $A_x = A\cos\theta$ and $A_y = A\sin\theta$

where $\vec{A}_x$ and $\vec{A}_y$ are two rectangular components of $\vec{A}$ along X-axis and Y-axis respectively and θ is the angle which $\vec{A}_x$ makes with $\vec{A}$.

- $\vec{A}=A_x\hat{i}+A_y\hat{j}$, $A=\sqrt{A_x^2+A_y^2}$ and $\frac{A_y}{A_x}=\tan\theta$

Projectile Fired at an Angle with the Horizontal

- Suppose a projectile is fired with velocity u at an angle θ with the horizontal. Let it reach the point (x, y) after time t. Then

(i) Components of initial velocity : $u_x = u\cos\theta$, $u_y = u\sin\theta$

(ii) Components of acceleration at any instant : $a_x = 0$, $a_y = -g$

(iii) Position after time t: $x=(u\cos\theta)t,\ y=(u\sin\theta)t-\frac{1}{2}gt^2$

(iv) Equation of trajectory : $y=x\tan\theta-\frac{g}{2u^2\cos^2\theta}x^2$

(v) Maximum height, $H=\frac{u^2\sin^2\theta}{2g}$

(vi) Time of flight, $T=\frac{2u\sin\theta}{g}$

(vii) Horizontal range, $R=\frac{u^2\sin2\theta}{g}$

(viii) Maximum horizontal range is attained at $\theta = 45°$ and its value is

$$R_{max}=\frac{u^2}{g}$$

(ix) Velocity after time t: $v_x = u\cos\theta$, $v_y = u\sin\theta - gt$

$\therefore\ v=\sqrt{v_x^2+v_y^2}$ and $\tan\beta=\frac{v_y}{v_x}$

Dot and Cross Product of Two Vectors

- **Dot Product :** $\vec{A}\cdot\vec{B}=AB\cos\theta$

where A and B are the magnitudes of two vectors $\vec{A}$ and $\vec{B}$ and θ is the angle between the two, the vectors being placed tail to tail.

- **Cross Product :** $\vec{A}\times\vec{B}=\vec{C}$, where $C=|\vec{A}\times\vec{B}|=AB\sin\theta$.

$\vec{C}$ is the cross-product of $\vec{A}$ and $\vec{B}$ and θ is the smaller angle between the two vectors when placed tail to tail. $\vec{C}$ is perpendicular to the plane containing $\vec{A}$ and $\vec{B}$

- $\cos\theta=\frac{\vec{A}\cdot\vec{B}}{AB}$, $\sin\theta=\frac{|\vec{A}\times\vec{B}|}{AB}$, $\tan\theta=\frac{|\vec{A}\times\vec{B}|}{\vec{A}\cdot\vec{B}}$

Relative Velocity and Relative Speed

- $\vec{v}_{AB}=\vec{v}_A-\vec{v}_B$, $\vec{v}_{BA}=\vec{v}_B-\vec{v}_A$, $\vec{v}_{AB}=-\vec{v}_{BA}$ and $v_{AB}=v_{BA}$

where $\vec{v}_A, \vec{v}_B$ = velocities of two bodies A and B, $\vec{v}_{AB}$ = relative velocity of A w.r.t. B, $\vec{v}_{BA}$ = relative velocity of B w.r.t. A, and $v_{AB} = v_{BA}$ = relative speed between A and B.

Projectile Motion on an Inclined Plane

SOLVED PAPER 2014

J&K-CET

Jammu & Kashmir Common Entrance Test

1. A solenoid of length 0.6 m has a radius of 1 cm and is made up of 600 turns. It carries a current of 6 A. What is the magnitude of the magnetic field inside the solenoid?
(a) 7.54×10^{-3} T (b) 10.84×10^{-3} T
(c) 4.87×10^{-3} T (d) 2.36×10^{-2} T

2. The bus moving with a speed of 42 km/hr is brought to a stop by brakes after 6 m. If the same bus is moving at a speed of 90 km/hr, then the minimum stopping distance is
(a) 15.48 m (b) 18.64 m
(c) 22.13 m (d) 27.55 m

3. A 15 pF capacitor is connected to a 60 V battery. How much electrostatic energy is stored?
(a) 2.77×10^{-8} joules (b) 1.77×10^{-8} joules
(c) 2.35×10^{-8} joules (d) 1.35×10^{-8} joules

4. In a video signal for transmission of picture, what value of bandwidth is used in communication system?
(a) 2.4 MHz (b) 4.2 MHz
(c) 24 MHz (d) 42 MHz

5. A straight wire of mass 300 g and length 2.5 m carries a current of 3.5 A. It is suspended in mid-air by a uniform horizontal magnetic field *B*. What is the magnitude of the magnetic field?
(a) 0.654 T (b) 0.336 T
(c) 1.576 T (d) 0.939 T

6. When longitudinal wave propagates, what happens in the region of compressions and rarefactions?
(a) Density varies
(b) Density remains constant
(c) There is heat transfers
(d) Boyle's law is obeyed

7. Calculate average thermal energy of the typical star if the temperature of the star is 12 million kelvins.
(a) 2.07×10^{-16} joules
(b) 1.07×10^{-16} joules
(c) 4.07×10^{-16} joules
(d) 3.07×10^{-16} joules

8. A galaxy is moving away from an observer on earth so that sodium light of wavelength 5892 Å is observed at 5896 Å. The speed of galaxy is
(a) 306 km/s (b) 204 km/s
(c) 185 km/s (d) 158 km/s

9. The correct order of arrangement of electromagnetic waves according to their wavelength is
(a) Gamma rays < Micro waves < AM radio waves < FM radio waves
(b) Micro waves < AM radio waves < FM radio wave < Gamma rays
(c) Gamma rays < AM radio waves < FM radio wave < Micro waves
(d) Gamma rays < Micro waves < FM radio waves < AM radio waves

10. A constant retarding force of 80 N is applied to a body of mass 50 kg which is moving initially with a speed of 20 m/s. What would be the time required by the body to come to rest?
(a) 15 s (b) 14 s (c) 12.5 s (d) 18 s

11. Light from a point source in air falls on a spherical glass surface (n = 1.67 and radius of curvature = 25 cm). The distance of the light source from the glass surface is 95 cm. At what position the image is formed?
(a) 75.45 cm (b) 90.50 cm
(c) 105.25 cm (d) 99.40 cm

12. The length of the wire is increase by 2% by applying a load of 2.5 kg-wt. What is the linear strain produced in the wire?
(a) 0.1 (b) 0.01 (c) 0.2 (d) 0.02

13. The half life of an old rock element is 5800 years. In how many years its sample of 25 gm is reduced to 6.25 gm.
(a) 2900 years (b) 5800 years
(c) 11600 years (d) 23200 years

14. The earth takes 24 hr to rotate once about its axis. How much time does the sun take to shift by 5° when viewed from the earth?
(a) 20 min (b) 15 min
(c) 10 min (d) 5 min

15. Which of the following is a dimensionless quantity?
(a) Magnetic flux density
(b) Electric flux density
(c) Lumen flux density
(d) Optical density

16. A crane lifts weight of 75 kg to a height of 15 m in 15 seconds. The power of the crane is (assuming $g = 9.8$ m/s^2)
(a) 635 watt (b) 735 watt
(c) 835 watt (d) 1135 watt

17. The escape velocity from the surface of earth is 11.2 km s^{-1}. What is the escape velocity in a planet whose radius is three times that of earth and on which the acceleration due to gravity is three times of that on earth?
(a) 11.2 km s^{-1} (b) 22.4 km s^{-1}
(c) 33.6 km s^{-1} (d) 5.6 km s^{-1}

18. What is the dimensional formula for work?
(a) $[MLT^{-2}]$ (b) $[ML^2T^{-2}]$
(c) $[ML^3T^{-2}]$ (d) $[ML^2T^{-1}]$

19. When the gas expands with temperature using the relation $V = KT^{2/3}$ for the temperature change of 40 K, the work done is
(a) $20.1R$ (b) $30.2R$
(c) $26.6R$ (d) $35.6R$

20. The resonant frequency of a *L-C-R* circuit depends upon
(a) L and R (b) C and R
(c) L and C (d) L, C and R

21. A wheel with 30 metallic spokes each of 0.7 m long is rotated with a speed of 120 rev/min, in a plane normal to the horizontal component of earth's magnetic field H_E at a place. If $H_E = 0.8$ G at the place, what is the induced emf between the axle and the rim of the wheel?
Given $1\text{ G} = 10^{-4}$ T
(a) 2.46×10^{-4} volt (b) 6.28×10^{-4} volt
(c) 5.76×10^{-5} volt (d) 4.92×10^{-4} volt

22. In a circuit, 3 resistors of resistances 1.2 Ω, 2 Ω, 3 Ω are connected in parallel. The value of equivalent resistance is
(a) Less than 1.2 Ω
(b) Greater than 1.2 Ω
(c) Between 1.2 Ω and 2 Ω
(d) Between 2 Ω and 3 Ω

23. The magnetic field is absent in which one of the following physical situation?
(a) Surface of Neutron star
(b) Surface of Earth
(c) Human nerve fiber
(d) At some point on the axis of bar magnet

24. What amount of energy is associated with mass of 2.5 kg?
(a) 6.27×10^{17} joules (b) 4.27×10^{17} joules
(c) 0.27×10^{17} joules (d) 2.27×10^{17} joules

25. The ratio of radii of nuclei of two atoms of elements of atomic mass numbers 27 and 64 is
(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9

26. MSI integrated chip contains
(a) Less than 10 gates (b) 10 to 100 gates
(c) 100 to 1000 gates (d) 1000 to 10000 gates

27. What is the torque of a force $3\hat{i} + 7\hat{j} + 4\hat{k}$ about the origin, if the force acts on a particle whose position vector is $2\hat{i} + 2\hat{j} + 1\hat{k}$?
(a) $\hat{i} - 5\hat{j} + 8\hat{k}$ (b) $2\hat{i} + 2\hat{j} + 2\hat{k}$
(c) $\hat{i} + \hat{j} + \hat{k}$ (d) $3\hat{i} + 2\hat{j} + 3\hat{k}$

28. A message signal of frequency 20 kHz and peak voltage of 15 volts is used to modulate a carrier

wave of 1000 kHz and peak voltage of 30 volts. The modulation index and upper side band are respectively

(a) 0.50 and 1020 kHz
(b) 0.66 and 1020 kHz
(c) 0.50 and 980 kHz
(d) 0.66 and 980 kHz

29. Calculate the efficiency of the engine if Carnot cycle operates at T_1 = 550 K and T_2 = 320 K producing 2.3 kJ of mechanical work per cycle?

(a) 0.418 (b) 0.622 (c) 0.823 (d) 0.902

30. Which of the following device is the application of Photoelectric effect?

(a) Light emitting diode
(b) Diode
(c) Photocell
(d) Transistor

31. The waves used for Line-of-Sight (LOS) communication are

(a) Ground waves (b) Space waves
(c) Sound waves (d) Sky waves

32. The inductance L of a solenoid depends upon its radius R as

(a) $L \propto R$ (b) $L \propto 1/R$
(c) $L \propto R^2$ (d) $L \propto R^3$

33. How much revolution does the engine make during the time when a motor wheel with angular speed is increased from 720 rpm to 2820 rpm in 14 seconds?

(a) 354 (b) 490 (c) 413 (d) 620

34. If the pressure at half of the lake is equal to 1/3 pressure at the bottom of the lake, what is the depth of the lake?
(Assume g = 10 m s^{-2} & 1 atm = 1 × 10^5 ; ρ water = 10^3)

(a) 9.6 m (b) 7.5 m (c) 4.4 m (d) 3.2 m

35. The value of Rydberg constant is

(a) 1.997×10^7 m^{-1} (b) 1.097×10^{-7} m^{-1}
(c) 1.097×10^7 m^{-1} (d) 19.97×10^7 m^{-1}

36. A car moving on a straight road covers 1/3 of the distance with 25 km/h and rest with 75 km/h. The average speed is

(a) 25 km/hr (b) 45 km/hr
(c) 55 km/hr (d) 75 km/hr

37. The distance at which average radius of the earth orbit subtends an angle of 1 arc second is

(a) Parsec (b) Astronomical unit
(c) Light year (d) Unified atomic unit

38. What happens to the acceleration due to gravity with the increase in altitude from the surface of the earth?

(a) Increases
(b) Decreases
(c) First decreases and then increases
(d) Remains same

39. Lenz's law is a consequence of the law of conservation of

(a) Charge (b) Mass
(c) Momentum (d) Energy

40. The angle between the true geographic north and the north shown by a compass needle is called as

(a) Inclination
(b) Magnetic Declination
(c) Angle of meridian
(d) Magnetic pole

41. An air bubble of 2 cm^3 rises from the bottom of a lake of 32 m at a temperature of 9°C. When the bubble reaches the surface of the lake from the bottom of the lake, what volume does it grows for which temperature is 30°C (Assume g = 10 m s^{-2} and density ρ = 10^3 kg m^{-3})

(a) 5.937 cm^3 (b) 8.937 cm^3
(c) 12.937 cm^3 (d) 16.937 cm^3

42. Which quantity is transmitted with propagation of longitudinal waves through a medium?

(a) Dispersion (b) Energy
(c) Matter (d) Frequency

43. A monochromatic light of frequency 3×10^{14} Hz is produced by a LASER, emits the power of 3×10^{-3} W. Find how many number of photons are emitted per second.

(a) 1.5×10^{16} (b) 2.5×10^{16}
(c) 4.5×10^{16} (d) 8.5×10^{16}

44. A mountaineer standing on the edge of a cliff 441 m above the ground throws a stone horizontally with an initial speed of 20 m s^{-1}. What is the speed with which the stone reaches the ground?
(a) 90 m s^{-1} (b) 95.08 m s^{-1}
(c) 85 m s^{-1} (d) 92 m s^{-1}

45. A zener diode is used as
(a) An amplifier
(b) A rectifier
(c) A voltage regulator
(d) A light emitting device

46. In which of the following does the intensity of sound vary with time?
(a) Doppler effect (b) Beats
(c) Transverse waves (d) Longitudinal waves

47. A solid cylinder of mass 30 kg rotates about its axis with an angular speed of 50 rad/s. What is the KE associated with the rotation of the cylinder, if the radius of the cylinder is 0.30 m?
(a) 1500.5 J (b) 1687.5 J
(c) 2000.7 J (d) 1350.0 J

48. The circuits that make rippled AC to pure DC are known as
(a) DC converters (b) Junction diodes
(c) Bipolar transistors (d) Filters

49. A string of mass 3 kg is under tension of 400 N. The length of the stretched string is 25 cm. If the transverse jerk is stuck at one end of the string how long does the disturbance take to reach the other end?
(a) 0.047 s (b) 0.055 s
(c) 0.034 s (d) 0.065 s

50. The total energy of an electron in 4^{th} orbit of hydrogen atom is
(a) −13.6 eV (b) −3.4 eV
(c) −1.51 eV (d) – 0.85 eV

51. The graph between voltage and current across a conductor that follows Ohm's law is
(a) Straight line (b) Parabolic
(c) Sine-curve (d) Cos-curve

52. An athlete throws the shot-put of mass 4 kg with initial speed of 2.2 m s^{-1} at 41° from a height of 1.3 m from the ground. What is the KE of the shot-put when it reaches the ground ? (Ignoring the air resistance and gravity $g = 9.8$ m s^{-2})
(a) 42.84 joules (b) 52.84 joules
(c) 62.84 joules (d) 72.84 joules

53. Two bulbs operating on standard voltage 110 volt have resistances in the ratio 9:16. The ratio of brightness of light from them is
(a) 9 : 16 (b) 16 : 9
(c) 3 : 4 (d) 4 : 3

54. The neutral point on potentiometer's scale for two cells of EMF 2.1 and E volts is observed at distances 40 cm and 56 cm respectively. The value of E is
(a) 2.10 volts (b) 2.94 volts
(c) 1.50 volts (d) 1.20 volts

55. Which one of the following equation is Torricelli law?
(a) $P = \rho gh$ (b) $v = \sqrt{2hg}$
(c) $\eta R_e = \rho v d$ (d) $S(2dl) = Fd$

56. Which of the law representing Maxwell's third equation?
(a) Gauss's law for electricity
(b) Ampere-Maxwell law
(c) Gauss's law of magnetism
(d) Faraday's law

57. A silver wire has a resistance of 1.6 Ω at 25.5 °C and a resistance of 2.5 Ω at 100°C, then temperature coefficient of resistivity of silver is
(a) 5.55×10^{-3} °C (b) 7.55×10^{-3} °C
(c) 11.75×10^{-2} °C (d) 15.5×10^{-3} °C

58. When a particle returns to its initial point, its
(a) Displacement is zero
(b) Average velocity is zero
(c) Distance is zero
(d) Average speed is zero

59. A body weight 45 N on the surface of the earth. What is the gravitational force on it due to the earth at a height equal to half of the radius of the earth?
(a) 20 N (b) 45 N (c) 40 N (d) 90 N

60. The angle of a prism is 42° and refractive index of its material is 3/2. Then angle of minimum deviation for this prism is

(a) 63° (b) 42° (c) 28° (d) 21°

61. A conducting sphere of radius 5 cm has an unknown charge. The electric field at 10 cm from the centre of the sphere is 1.8×10^3 N/C and points radially inward. What is the net charge on the sphere?

(a) 1.8 nC (b) 2 nC
(c) 1 nC (d) 1.5 nC

62. For transistor the value of β is 50 then the value of α is

(a) 50/51 (b) 49/50 (c) 51/50 (d) 50/49

63. An infinite line charge produces a field of 18×10^5 N/C at a distance of 4 cm. What is the linear charge density?

(a) 18 μC/m (b) 5 μC/m
(c) 4 μC/m (d) 10 μC/m

64. What is the fraction of molecular volume to the actual volume occupied by oxygen gas at STP? Given the diameter of oxygen molecule is 2 Å.

(a) 1.75×10^{-4} (b) 1.5×10^{-4}
(c) 12×10^{-4} (d) 1.125×10^{-4}

65. A 400 pF capacitor is charged by a 100 V supply. How much electrostatic energy is lost in the process of disconnecting from the supply and connecting another uncharged 400 pF capacitor?

(a) 10^{-5} joules (b) 10^{-6} joules
(c) 10^{-7} joules (d) 10^{-4} joules

66. The ratio of magnitude of electric displacement (D) and electric field (E) is

(a) Charge density
(b) Permittivity
(c) Electric susceptibility
(d) Dielectric constant

67. An object is gently placed on a long converges belt moving with 11 m s^{-1}. If the coefficient of friction is 0.4, then the block will slide in the belt up to distance of

(a) 10.21 m (b) 15.43 m
(c) 20.3 m (d) 25.6 m

68. A spring balance has a scale that reads from 0 to 60 kg. The length of the scale is 30 cm. A body suspended from this balance and when displaced and released, oscillates with a period of 0.8 s, what is the weight of the body when oscillating?

(a) 350.67 N (b) 540.11 N
(c) 311.24 N (d) 300.5 N

69. The magnetic needle has magnetic moment 8.7×10^{-2} A m^2 and moment of inertia 11.5×10^{-6} kg m^2. It performs 10 complete oscillations in 6.70 s, what is the magnitude of the magnetic field?

(a) 0.012 T (b) 0.120 T
(c) 1.200 T (d) 2.10 T

70. The distance for which ray optics is good approximation for an aperture 5 mm and wavelength 5000 Å is

(a) 18 meter (b) 30 meter
(c) 38 meter (d) 50 meter

71. A compound microscope has a magnification of 30. The focal length of the eyepiece is 5 cm. If the final image is formed at the least distance of distinct vision (25 cm), the magnification produced by the objective is

(a) 10 (b) 7.5 (c) 5 (d) 15

72. Which of the following relation is called mirror equation?

(a) $\frac{u}{v}+\frac{f}{u}=\frac{1}{f}$ (b) $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

(c) $\frac{1}{f}+u=\frac{1}{v}$ (d) $uf+vf=uv$

73. A stone of mass 0.05 kg is thrown vertically upwards. What is the direction and magnitude of the net force on the stone during its upward motion?

(a) 0.49 N vertically downwards direction
(b) 0.49 N vertically upwards direction
(c) 0.05 N vertically downwards direction
(d) 9.8 N vertically upwards direction

74. Two different coils have self-inductance $L_1 = 8$ mH, $L_2 = 2$ mH. The current in one coil is increased at a constant rate. The current in the

second coil is also increased at the same rate. At a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are i_1, V_1, and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2, and W_2 respectively. Then,

(a) $\frac{W_2}{W_1} = 8$ (b) $\frac{W_2}{W_1} = \frac{1}{8}$

(c) $\frac{W_2}{W_1} = 4$ (d) $\frac{W_2}{W_1} = \frac{1}{4}$

75. A concave mirror of focal length f produces a real image n times the size of the object. The distance of the object from the mirror is

(a) $(n-1)f$ (b) $(n+1)f$

(c) $\frac{(n+1)f}{n}$ (d) $\frac{(n-1)f}{n}$

SOLUTIONS

1. (a) : Here,

Length of the solenoid, $l = 0.6$ m

Number of turns, $N = 600$

Radius, $r = 1$ cm $= 10^{-2}$ m

Current in the solenoid, $I = 6$ A

Number of turns per unit length is

$$n = \frac{N}{l} = \frac{600}{0.6} = 1000 \text{ turns/m}$$

As $\frac{l}{r} = \frac{0.6}{10^{-2}} = 60$ *i.e.* $l >> r$

Hence, we can use the formula for magnetic field inside a long solenoid.

i.e. $B = \mu_0 nI = 4\pi \times 10^{-7} \times 1000 \times 6 = 7.54 \times 10^{-3}$ T

2. (d) : For the first case,

Initial speed of the bus,

$$u = 42 \text{ km/hr} = 42 \times \frac{5}{18} \text{ m/s} = \frac{35}{3} \text{ m/s}$$

Final speed of the bus, $v = 0$

Distance travelled by the bus before it stops,

$s = 6$ m

As $v^2 - u^2 = 2as$

$$\therefore \quad (0)^2 - \left(\frac{35}{3}\right)^2 = 2 \times a \times 6$$

$$a = -\frac{1225}{108} \text{ m/s}^2$$

For the second case,

$$u = 90 \text{ km/hr} = 90 \times \left(\frac{5}{18}\right) \text{m/s} = 25 \text{ m/s}$$

$$v = 0,\ a = -\frac{1225}{108} \text{ m/s}^2,\ s = ?$$

$$\therefore \quad (0)^2 - (25)^2 = 2\left(-\frac{1225}{108}\right)s$$

$$s = \frac{25 \times 25 \times 108}{2 \times 1225} \text{ m} = 27.55 \text{ m}$$

3. (a) : Here, $C = 15$ pF $= 15 \times 10^{-12}$ F

$V = 60$ V

Electrostatic energy stored in the capacitor is

$$U = \frac{1}{2}CV^2$$

$$= \frac{1}{2} \times 15 \times 10^{-12} \times (60)^2 = 2.7 \times 10^{-8} \text{J}$$

4. (b) : A video signal of bandwidth 4.2 MHz is used for transmission of picture.

5. (b) : Here,

Mass of the wire, $m = 300$ g $= 300 \times 10^{-3}$ kg

$= 0.3$ kg

Length of the wire, $l = 2.5$ m

Current in the wire, $I = 3.5$ A

For mid-air suspension,

Magnetic force on the wire = Weight of the wire

$IlB \sin 90° = mg$

$$B = \frac{mg}{Il} = \frac{0.3 \times 9.8}{3.5 \times 2.5} = 0.336 \text{ T}$$

6. (a) : In a longitudinal wave, regions of high density correspond to compressions and the regions of low density correspond to rarefactions.

Thus, when longitudinal wave propagates density varies in the region of compressions and rarefactions.

7. (*) : Here,

Temperature of the star, $T = 12$ million K

$= 12 \times 10^6$ K

Boltzmann constant, $k_B = 1.38 \times 10^{-23}$ J K^{-1}

Average thermal energy of the star is

$$\bar{E} = \frac{3}{2}k_B T = \frac{3}{2}\times 1.38\times 10^{-23}\times 12\times 10^6$$

$= 2.484 \times 10^{-16}$ J

* None of the given options is correct.

8. (b) : Here,

$\Delta\lambda = 5896$ Å – 5892 Å = 4 Å

$\lambda = 5892$ Å

As $\dfrac{\Delta\lambda}{\lambda} = \dfrac{v}{c}$

$$\therefore \quad v = \frac{\Delta\lambda}{\lambda}c = \frac{4}{5892}\times 3\times 10^8 \text{ m/s}$$

$= 2.04 \times 10^5$ m/s = 204 km/s

Hence, the speed of galaxy is 204 km/s.

9. (d) : According to their wavelength, the correct order of given electromagnetic waves is

Gamma rays < Micro waves < FM radio waves < AM radio waves

10. (c) : Here,

Retarding force, $F = 80$ N

Mass of the body, $m = 50$ kg

Initial speed of the body, $u = 20$ m/s

As retarding force = Mass × Retardation

$$\therefore \quad \text{Retardation} = \frac{F}{m} = \frac{80 \text{ N}}{50 \text{ kg}} = 1.6 \text{ m/s}^2$$

Let t be time required by the body to come to rest.

$$\therefore \quad t = \frac{v-u}{a} = \frac{0-20}{-1.6} = 12.5 \text{ s}$$

11. (*) : Here,

$u = -95$ cm, $n_1 = 1$, $n_2 = 1.67$, $R = +25$ cm

As light travels from air (rarer medium) to glass (denser medium), so

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1.67}{v} - \frac{1}{-95} = \frac{1.67-1}{25}$$

$$\frac{1.67}{v} + \frac{1}{95} = \frac{0.67}{25}$$

$$\frac{1.67}{v} = \frac{0.67}{25} - \frac{1}{95}$$

$$\frac{1.67}{v} = \frac{12.73-5}{475} = \frac{7.73}{475}$$

$$v = \frac{475\times 1.67}{7.73} = +102.62 \text{ cm}$$

The image is formed at a distance of 102.62 cm from the glass surface in the direction of incident light.

* None of the given options is correct.

12. (d) : Let L be the original length of the wire. Increase in length

$$L' = L + \frac{2}{100}L = \frac{102}{100}L = 1.02L$$

$\therefore$ Change in length, $\Delta L = L' - L = 1.02L - L = 0.02L$

$$\text{Strain} = \frac{\Delta L}{L} = \frac{0.02L}{L} = 0.02$$

13. (c): Here,

Half life, $T_{1/2} = 5800$ years

Amount of sample left after n half lives is

$$N = N_0\left(\frac{1}{2}\right)^n$$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

$$\frac{6.25 \text{ gm}}{25 \text{ gm}} = \left(\frac{1}{2}\right)^n$$

$$\frac{1}{4} = \left(\frac{1}{2}\right)^n \text{ or } \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^n$$

$n = 2$

$\therefore \quad t = n\,T_{1/2} = 2 \times 5800$ years = 11600 years

14. (a) : Time taken for 360° shift = 24 h

Time taken for 1° shift

$$= \frac{24}{360}\text{ h} = \frac{24}{360}\times 60 \text{ min} = 4 \text{ min}$$

$\therefore$ Time taken for 5° shift = 4 × 5 min = 20 min

15. (d) : Optical density is the ratio of the speed of light in two media. As optical density is the ratio of two similar physical quantities, therefore it is the dimensionless quantity.

All other given physical quantities have dimensions.

16. (b) : Here,

$m = 75$ kg, $h = 15$ m, $t = 15$ s

Power of the crane is

$$P = \frac{W}{t} = \frac{mgh}{t} = \frac{75\times9.8\times15}{15} = 735 \text{ watt}$$

17. (c): Here,

Radius of the earth = R_e

Radius of the planet, $R_p = 3R_e$

Acceleration due to gravity on the surface of earth = g_e

Acceleration due to gravity on the surface of planet, $g_p = 3g_e$

Escape velocity from the surface of the earth is

$$v_e = \sqrt{2g_e R_e} = 11.2 \text{ km s}^{-1}$$

Escape velocity from the surface of the planet is

$$v_p = \sqrt{2g_p R_p} = \sqrt{2(3g_e)(3R_e)}$$

$$= 3\sqrt{2g_e R_e} = 3\times11.2 \text{ km s}^{-1} = 33.6 \text{ km s}^{-1}$$

18. (b) : Work = Force $\times$ Distance

$$= [\mathrm{MLT^{-2}}][\mathrm{L}] = [\mathrm{ML^2T^{-2}}]$$

19. (c): Given :

$$V = KT^{2/3} \qquad \text{...(i)}$$

Differentiating both sides,

$$\Delta V = \frac{2}{3}KT^{-1/3}\Delta T \quad (\text{As } K \text{ is a constant}) \qquad \text{...(ii)}$$

According to ideal gas equation for 1 mole

$$PV = RT$$

$$P = \frac{RT}{V} \qquad \text{...(iii)}$$

Work done by the gas is

$W = P\Delta V$

$$= \frac{RT}{V}\frac{2}{3}KT^{-1/3}\Delta T \qquad (\text{Using (ii) and (iii)})$$

$$= \frac{2}{3}\frac{RKT^{2/3}}{V}\Delta T$$

$$= \frac{2}{3}\frac{RKT^{2/3}}{KT^{2/3}}\Delta T \qquad (\text{Using (i)})$$

$$= \frac{2}{3}R\Delta T = \frac{2}{3}\times R\times40 = 26.6R$$

20. (c): The resonant frequency of a *L*-*C*-*R* circuit is

$$\upsilon_r = \frac{1}{2\pi\sqrt{LC}}$$

Above expression shows that it depends upon *L* and *C*.

21. (a) : Here,

Radius of the wheel, $R = 0.7$ m

Frequency of rotation of the wheel,

$$\upsilon = 120 \text{ rev/min} = \frac{120}{60}\text{ rev/sec} = 2 \text{ rev/sec}$$

Magnetic field, $B = H_E = 0.8\text{ G} = 0.8\times10^{-4}$ T

Induced emf across the ends of a spoke,

$$\varepsilon = B\pi R^2\upsilon$$

$$= 0.8\times10^{-4}\times3.14\times(0.7)^2\times2$$

$$= 2.46\times10^{-4}\text{ V}$$

As all the 30 spokes are connected in parallel, therefore emf induced across each spoke is same. Thus, the induced emf between the axle and the rim of the wheel is same as across any spoke *i.e.* 2.46×10^{-4} V.

The number of spokes is immaterial.

22. (a) : Three resistors of resistances 1.2 Ω, 2 Ω and 3 Ω are connected in parallel. Their equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{1.2}+\frac{1}{2}+\frac{1}{3} = \frac{10}{12}+\frac{1}{2}+\frac{1}{3}$$

$$= \frac{10+6+4}{12} = \frac{20}{12} = \frac{5}{3}$$

$$R_{eq} = \frac{3}{5} = 0.6\ \Omega$$

Thus, the value of equivalent resistance is less than 1.2 Ω.

23. (*) : Magnetic field is present in all the given physical situations.

* None of the given options is correct.

24. (d) : Here,

Mass, $m = 2.5$ kg

According to Einstein mass-energy equivalence relation

$E = mc^2$ where *c* is the velocity of light in vacuum.

$E = (2.5\text{ kg})(3\times10^8\text{ m s}^{-1})^2$

$= 22.5\times10^{16}\text{ J} = 2.25\times10^{17}\text{ J}$

25. (a) : Nuclear radius, $R = R_0(A)^{1/3}$
where R_0 is a constant and A is the atomic mass number.

$$\therefore \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{27}{64}\right)^{1/3} = \left(\frac{3^3}{4^3}\right)^{1/3} = \frac{3}{4}$$

26. (b) : Depending upon the number of circuit components or logic gates, ICs (integrated circuits) are termed as in table.

S. No.	Term used	Number of logic gates
1.	Small Scale Integration (SSI)	≤ 10
2.	Medium Scale Integration (MSI)	≤ 100
3.	Large Scale Integration (LSI)	≤ 1000
4.	Very Large Scale Integration(VLSI)	> 1000

27. (a) : Here, $\vec{r} = 2\hat{i} + 2\hat{j} + 1\hat{k}$

$$\vec{F} = 3\hat{i} + 7\hat{j} + 4\hat{k}$$

Torque, $\vec{\tau} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 3 & 7 & 4 \end{vmatrix}$$

$$= \hat{i}(8-7) - \hat{j}(8-3) + \hat{k}\,(14-6)$$

$$= \hat{i} - 5\hat{j} + 8\hat{k}$$

28. (a) : Here,

$A_m = 15$ V, $A_c = 30$ V

$\upsilon_m = 20$ kHz, $\upsilon_c = 1000$ kHz

Modulation index, $\mu = \dfrac{A_m}{A_c} = \dfrac{15 \text{ V}}{30 \text{ V}} = 0.50$

Upper side band at $(\upsilon_m + \upsilon_c) = (20 + 1000)$ kHz $= 1020$ kHz

29. (a) : Here, $T_1 = 550$ K and $T_2 = 320$ K

Efficiency of the Carnot engine, $\eta = 1 - \dfrac{T_2}{T_1}$

$$\therefore \quad \eta = 1 - \frac{320}{550} = \frac{550-320}{550} = \frac{230}{550} = 0.418$$

30. (c) : A photocell is a technological application of the photoelectric effect.

31. (b) : Space waves are used for Line-of-Sight (LOS) communication.

32. (c) : The inductance of a solenoid is

$$L = \mu_0 n^2 Al$$

where A is the area of cross-section of the solenoid, l its length and n is the number of turns per unit length.

As $A = \pi R^2$

where R is the radius of the solenoid.

$\therefore \quad L = \mu_0 n^2 \pi R^2 l$

$L \propto R^2$

33. (c) : Here,

Initial angular speed of the engine,

$$\omega_0 = 720 \times \frac{2\pi}{60} \text{ rad/s} = 24\pi \text{ rad/s}$$

Final angular speed of the engine,

$$\omega = 2820 \times \frac{2\pi}{60} \text{ rad/s} = 94\pi \text{ rad/s}$$

Time in which the change in speed takes place,

$t = 14$ s

Angular acceleration of the engine,

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{94\pi - 24\pi}{14}$$

$$= \frac{70\pi}{14} \text{rad/s}^2 = 5\pi \text{ rad/s}^2$$

The angular displacement in time t is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$= (24\pi \times 14) + \frac{1}{2} \times 5\pi \times (14)^2$$

$$= (336\pi + 490\pi) \text{ rad} = 826\pi \text{ rad}$$

Number of revolutions made by the engine in 14 s

$$= \frac{\theta}{2\pi} = \frac{826\pi}{2\pi} = 413$$

34. (*): Pressure of bottom of the lake $= P_0 + h\rho g$

Pressure at half depth of lake $= P_0 + \dfrac{h}{2\rho g}$

According to question,

$$P_0 + \frac{h}{2}\rho g = \frac{1}{3}(P_0 + h\rho g)$$

$$\Rightarrow \quad P_0 - \frac{1}{3}P_0 = \frac{1}{3}h\rho g - \frac{h}{2}\rho g$$

or $h = \frac{-4 \times P_0}{\rho g} = \frac{-4 \times 10^5}{10^3 \times 10} = -40$ m

Note : There is negative sign in answer which is not given in options. Hence, non of the given option is correct.

35. (c) : The value of Rydberg constant is 1.097×10^7 m^{-1}.

36. (b) : Let S km be total distance covered by the car.

Time taken by the car to cover $S/3$ with speed 25 km/h is

$$t_1 = \frac{S/3}{25} \text{ h}$$

and time taken by the car to cover remaining $\frac{2}{3}S$ with speed 75 km/h is

$$t_2 = \frac{2S/3}{75} \text{ h}$$

Average speed of the car is

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

$$= \frac{S}{t_1 + t_2} = \frac{S}{\left(\frac{S/3}{25}\right) + \left(\frac{2S/3}{75}\right)}$$

$$= \frac{S}{\frac{S}{75} + \frac{2S}{225}} = \frac{S}{\frac{3S + 2S}{225}} = \frac{225S}{5S} = 45 \text{ km/h}$$

37. (a) : Parsec is the distance at which average radius of earth's orbit substends an angle of 1 arc second.

1 parsec = 3.08×10^6 m

38. (b) : Acceleration due to gravity at a altitude h from the surface of the earth is

$$g_h = \frac{gR_E}{(R_E + h)^2} \quad \text{... (i)}$$

where g is the acceleration due to gravity on the surface of the earth and R_E is the radius of the earth.

Eq. (i) shows that acceleration due to gravity decreases with the increase in altitude.

39. (d) : Lenz's law is a consequence of the law of conservation of energy.

40. (b) : The angle between the true geographic north and the north shown by a compass needle is called as magnetic declination or simply declination.

41. (b) : Here,

Atmospheric pressure, P_0 = 1 atm

$= 1.01 \times 10^5$ Pa

Temperature at the bottom of the lake,

T_1 = 9°C = (273 + 9) K = 282 K

Volume of the air bubble at the bottom of the lake, V_1 = 2 cm^3

Pressure at the bottom of the lake,

$P_1 = P_0 + \rho g h$

$= 1.01 \times 10^5 + 10^3 \times 10 \times 32$ Pa

$= 1.01 \times 10^5 + 3.2 \times 10^5$ Pa $= 4.21 \times 10^5$ Pa

Pressure at the surface of the lake,

$P_2 = P_0 = 1.01 \times 10^5$ Pa

Temperature at the surface of the lake,

T_2 = 30°C = (273 + 30) K = 303 K

Let V_2 be volume of the air bubble at the surface of the lake.

As $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$\therefore \quad V_2 = \frac{P_1}{P_2}\frac{T_2}{T_1}V_1$$

$$= \left(\frac{4.21 \times 10^5 \text{ Pa}}{1.01 \times 10^5 \text{ Pa}}\right)\left(\frac{303 \text{ K}}{282 \text{ K}}\right)(2 \text{ cm}^3)$$

$= 8.621$ cm^3

42. (b) : Propagation of longitudinal waves through a medium leads to transmission of energy through the medium.

43. (a) : Energy of a photon, $E = h\upsilon$

$E = (6.63 \times 10^{-34}$ ϑ σ) $(3 \times 10^{14}$ Hζ)

$= 19.89 \times 10^{-20}$ J

Power of the source, $P = 3 \times 10^{-3}$ W

Number of photons emitted per second,

$$N = \frac{P}{E} = \frac{3 \times 10^{-3} \text{ W}}{19.89 \times 10^{-20} \text{ J}} = 1.5 \times 10^{16}$$

44. (b) : Here,

Height of the cliff, h = 441 m

Initial speed with which the stone is thrown,

$u = 20$ m/s

Let v be the speed with which the stone reaches the ground. Then

$v^2 - u^2 = 2gh$

$$v = \sqrt{u^2 + 2gh} = \sqrt{(20)^2 + 2\times 9.8\times 441} = 95.09 \text{ m/s}$$

45. (c) : A zener diode is used as a voltage regulator.

46. (b) : Beats is the phenomenon of regular variation in the intensity of sound with time when two sources of nearly equal frequencies are sounded together. One minimum of sound followed by one maximum constitute one beat.

47. (b) : Here,

Mass of the cylinder, $M = 30$ kg

Angular speed of rotation, $\omega = 50$ rad/s

Radius of the cylinder, $R = 0.3$ m

Moment of inertia of the cylinder about its axis

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\times 30\times (0.3)^2 = 1.35 \text{ kg m}^2$$

Rotational kinetic energy of the cylinder

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\times 1.35\times (50)^2 = 1687.5 \text{ J}$$

48. (d) : Filters are the circuits that make rippled AC to pure DC.

49. (a) : Here,

Mass of the string, $m = 3$ kg

Length of the string, $L = 25 \text{ cm} = 25\times 10^{-2}$ m

Tension in the string, $T = 400$ N

Mass per unit length of the string,

$$\mu = \frac{m}{L} = \frac{3 \text{ kg}}{25\times 10^{-2} \text{ m}} = 12 \text{ kg m}^{-1}$$

Speed of the wave on the string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{400 \text{ N}}{12 \text{ kg m}^{-1}}} = 5.77 \text{ m s}^{-1}$$

Time taken by disturbance to reach the other end

$$t = \frac{L}{v} = \frac{25\times 10^{-2} \text{ m}}{5.77 \text{ m s}^{-1}} = 0.043 \text{ s}$$

50. (d) : The total energy of an electron in n^{th} orbit of hydrogen atom is

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

For fourth orbit, $n = 4$

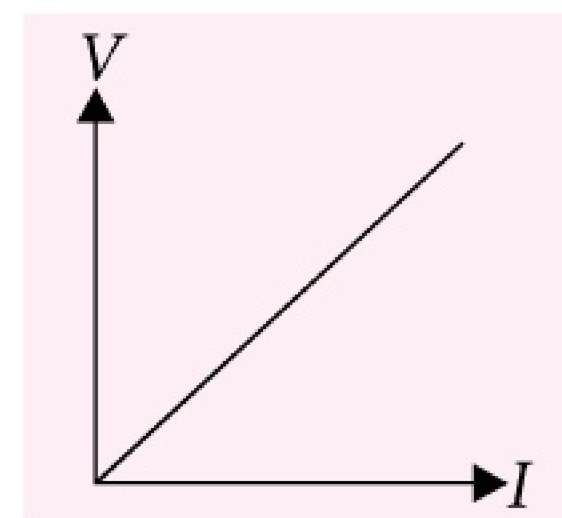
$$\therefore \quad E_4 = -\frac{13.6}{4^2} = -0.85 \text{ eV}$$

51. (a) : According to Ohm's law

$V = RI$

where R is the resistance of the conductor.

Hence, the graph between voltage V and current I is a straight line as shown in the figure.



52. (*): Here,

Mass of the shot-put, $m = 4$ kg

Initial speed with which the shot-put is thrown,

$u = 2.2 \text{ m s}^{-1}$

Initial kinetic energy of the shot-put

$$= \frac{1}{2}mu^2 = \frac{1}{2}\times (4 \text{ kg})(2.2 \text{ m s}^{-1})^2 = 9.68 \text{ J}$$

Initial potential energy of the shot-put at a height 1.3 m above ground

$= mgh = (4 \text{ kg})\,(9.8 \text{ m s}^{-2})\,(1.3 \text{ m}) = 50.96$ J

Total initial energy of the shot-put

$= 9.68 \text{ J} + 50.96 \text{ J} = 60.64$ J

Final potential energy of the shot-put as it hits the ground ($h = 0$) = zero

As the air resistance is negligible, therefore kinetic energy of the shot-put on hitting the ground = 60.64 J

* None of the given options in correct.

53. (b) : Here, $\frac{R_1}{R_2} = \frac{9}{16}$

Rate of production of heat,

$$P = \frac{V^2}{R}$$

For a given V,

$$P \propto \frac{1}{R}$$

$$\therefore \quad \frac{P_1}{P_2} = \frac{R_2}{R_1} = \frac{16}{9}$$

54. (b) : Here,

$E_1 = 2.1\,\text{V}, \quad E_2 = E$

$l_1 = 40$ cm, $l_2 = 56$ cm

As $\dfrac{E_1}{E_2} = \dfrac{l_1}{l_2}$

where E_1 and E_2 are the emfs of two cells and l_1, l_2 are the corresponding balancing lengths of the potentiometer wire.

$\therefore \quad \dfrac{2.1}{E} = \dfrac{40}{56}$ or $E = \dfrac{56}{40} \times 2.1 = 2.94\,\text{V}$

55. (b) : Speed of efflux, $v = \sqrt{2hg}$

where h is the depth of orifice below the liquid surface.

This equation is known as Torricelli law.

56. (d) : Maxwell's third equation is

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\phi_B}{dt}$$

It represents Faraday's law.

57. (*): Here,

$T_1 = 25.5°\text{C}, \quad T_2 = 100°\text{C}$

$R_{T_1} = 1.6\ \Omega, \quad R_{T_2} = 2.5\ \Omega$

Temperature coefficient of resistivity

$$\alpha = \frac{R_{T_2} - R_{T_1}}{R_{T_1}(T_2 - T_1)} = \frac{2.5 - 1.6}{1.6(100 - 25.5)}$$

$$= \frac{0.9}{1.6 \times 74.5} = 7.55 \times 10^{-3}\ °\text{C}^{-1}$$

* None of the given options is correct.

Note : In the options the unit of α is given as °C. On this basis none of the options is correct. The correct unit of α is $°\text{C}^{-1}$.

58. (a,b): When a particle returns to its initial position its displacement is zero.

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time taken}}$$

As displacement is zero, therefore its average velocity is also zero.

59. (a) : Weight of the body on the surface of the earth = mg = 45 N

Acceleration due to gravity at a height h from the surface of the earth is

$$g_h = \frac{gR_E^2}{(R_E + h)^2}$$

where R_E is the radius of the earth.

As $h = \dfrac{R_E}{2}$

$$\therefore \quad g_h = \frac{gR_E^2}{\left(R_E + \frac{R_E}{2}\right)^2} = \frac{4}{9}g$$

Gravitational force on the body at height h is

$$F = mg_h = m \times \frac{4}{9}g = \frac{4}{9} \times mg = \frac{4}{9} \times 45\text{ N} = 20\text{ N}$$

60. (d) : Here,

Angle of prism, $A = 42°$

Refractive index, $\mu = \dfrac{3}{2}$

$$\text{As} \quad \mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

where δ_m is the angle of minimum deviation.

$$\sin\left(\frac{A + \delta_m}{2}\right) = \mu \sin\left(\frac{A}{2}\right)$$

$$\frac{A + \delta_m}{2} = \sin^{-1}\left(\mu \sin\left(\frac{A}{2}\right)\right)$$

$$\delta_m = 2\sin^{-1}\left(\mu \sin\frac{A}{2}\right) - A$$

$$= 2\sin^{-1}\left(\frac{3}{2}\sin\frac{42°}{2}\right) - 42° = 23°$$

61. (b) : Here,

Radius of sphere, $R = 5$ cm $= 5 \times 10^{-2}$ m

Distance of point from the centre of the sphere,

$r = 10$ cm $= 10 \times 10^{-2}$ m

Electric field, $E = 1.8 \times 10^3$ N/C

$$\text{As} \quad E = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2} \qquad (\text{For } r > R)$$

$$\therefore \quad |q| = 4\pi\varepsilon_0 Er^2 = \frac{Er^2}{\frac{1}{4\pi\varepsilon_0}}$$

$$= \frac{(1.8 \times 10^3\text{ N/C})(10 \times 10^{-2}\text{ m})^2}{9 \times 10^9\text{ N m}^2/\text{C}^2}$$

$$= 2 \times 10^{-9}\text{ C} = 2\text{ nC}$$

As the electric field points radially inward, therefore the charge on the sphere is negative *i.e.* $q = -2$ nC.

62. (a) : Here, $\beta = 50$

The relation between α and β is

$$\alpha = \frac{\beta}{1+\beta} = \frac{50}{1+50} = \frac{50}{51}$$

63. (c): Here,

$E = 18 \times 10^5$ N/C, $r = 4$ cm $= 4 \times 10^{-2}$ m

$$\text{As} \quad E = \frac{\lambda}{2\pi\varepsilon_0 r} \quad \therefore \ \lambda = 2\pi\varepsilon_0 Er = \frac{Er}{2\left(\frac{1}{4\pi\varepsilon_0}\right)}$$

$$= \frac{(18\times10^5 \text{ N/C})(4\times10^{-2}\text{ m})}{2\times9\times10^9 \text{ N m}^2/\text{C}^2}$$

$$= 4 \times 10^{-6} \text{ C} = 4\ \mu\text{C/m}$$

64. (d) : Here,

Diameter of oxygen molecule, $d = 2$ Å

Radius of oxygen molecule,

$$r = \frac{d}{2} = \frac{2}{2}\text{ Å} = 1\text{ Å} = 10^{-10}\text{ m}$$

$$\text{Molecular volume} = \frac{4}{3}\pi r^3 N_A$$

where N_A is Avogadro's number

$$= \frac{4}{3}\times3.14\times(10^{-10})^3\times6.023\times10^{23}$$

$$= 2.52 \times 10^{-6}\text{ m}^3$$

Actual volume occupied by 1 mole of oxygen at STP

$$= 22.4 \text{ litre} = 22.4 \times 10^{-3}\text{ m}^3$$

$$\therefore \quad \frac{\text{Molecular volume}}{\text{Actual volume}} = \frac{2.52\times10^{-6}\text{ m}^3}{22.4\times10^{-3}\text{ m}^3}$$

$$= 1.125 \times 10^{-4}$$

65. (b) : Here,

$C_1 = 400$ pF $= 400 \times 10^{-12}$ F

$C_2 = 400$ pF $= 400 \times 10^{-12}$ F

$V_1 = 100$ V

Initial energy stored in capacitor C_1 is

$$U_i = \frac{1}{2}C_1V_1^2 = \frac{1}{2}\times400\times10^{-12}\times(100)^2$$

$$= 2 \times 10^{-6}\text{ J}$$

When capacitor C_1 is connected to an uncharged charged capacitor C_2, the charge flows and both attains a common potential V.

$$V = \frac{\text{Total charge}}{\text{Total capacitance}} = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1+0}{C_1+C_2}$$

$$= \frac{400\times10^{-12}\text{ F}\times100\text{ V}}{(400+400)\times10^{-12}\text{ F}} = 50\text{ V}$$

Final energy stored in capacitors C_1 and C_2 is

$$U_f = \frac{1}{2}(C_1+C_2)V^2$$

$$= \frac{1}{2}\times(400\times10^{-12}+400\times10^{-12})\times(50)^2$$

$$= \frac{1}{2}\times800\times10^{-12}\times(50)^2 = 10^{-6}\text{ J}$$

66. (b) : The ratio of magnitudes of electric displacement (D) and electric field (E) is

$$\frac{D}{E} = \varepsilon_0 K = \varepsilon$$

where ε is the permittivity of the medium.

67. (b) : Here, $u = 11$ m s^{-1}, $\mu = 0.4$

Force of friction, $f = \mu mg$

$$\therefore \quad \text{Retardation due to friction} = \frac{F}{m} = \frac{\mu mg}{m}$$

$$= \mu g = 0.4 \times 9.8 = 3.92 \text{ m s}^{-2}$$

Using $v^2 - u^2 = 2as$

$(0)^2 - (11)^2 = 2(-3.92)s$ or $s = 15.43$ m

68. (c): The 30 cm length of the scale reads upto 60 kg.

$\therefore$ Maximum force, $F = 60$ kg wt $= 60 \times 9.8$ N
$= 588$ N

and maximum extension, $x = 30 - 0 = 30$ cm
$= 30 \times 10^{-2}$ m

Spring constant of the spring balance is

$$k = \frac{F}{x} = \frac{588\text{ N}}{30\times10^{-2}\text{ m}} = 1960\text{ N/m}$$

Let a body of mass m is suspended from this balance.

Then, Time period of oscillation,

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{or} \quad T^2 = \frac{4\pi^2 m}{k}$$

$$\therefore \quad m = \frac{T^2 k}{4\pi^2} = \frac{(0.8)^2\times(1960)}{4\times(3.14)^2} = 31.8\text{ kg}$$

Weight of the body = mg = (31.8 kg) (9.8 m/s^2)
= 311.64 N

69. (a) : Here,
Magnetic moment, $M = 8.7 \times 10^{-2}$ A m^2
Moment of inertia, $I = 11.5 \times 10^{-6}$ kg m^2
Time period of oscillation is

$$T = \frac{6.70}{10} = 0.675 \text{ s}$$

As $T = 2\pi\sqrt{\frac{I}{MB}}$

$$\therefore \quad B = \frac{4\pi^2 I^2}{MT^2} = \frac{4\times(3.14)^2\times 11.5\times 10^{-6}}{8.7\times 10^{-2}\times(0.67)^2} = 0.012 \text{ T}$$

70. (d) : Here, a = 5 mm = 5×10^{-3} m
λ = 5000 Å = 5000×10^{-10} m
Fresnel distance (distance for which ray optics is a good approximation),

$$Z_F = \frac{a^2}{\lambda} = \frac{(5\times 10^{-3}\text{m})^2}{5000\times 10^{-10}\text{m}} = 50 \text{ m}$$

71. (c): Here,
Magnification of the compound microscope, $m = 30$
Focal length of the eyepiece, $f_e = 5$ cm
Least distance of distinct vision, D = 25 cm
When the final image is formed at D, the magnification due to the eyepiece is

$$m_e = \left(1 + \frac{D}{f_e}\right) = \left(1 + \frac{25}{5}\right) = 6$$

As $m = m_o m_e$
$\therefore$ Magnification produced by the objective is

$$m_o = \frac{m}{m_e} = \frac{30}{6} = 5$$

72. (b, d) : The relation between object distance (u), image distance (v) and focal length (f) of the mirror is

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \qquad \text{...(i)}$$

This relation is known as the mirror equation.
Eqn. (i) can be written as
$uf + vf = uv$
Hence, option (a) is also correct.

73. (a) : Here, Mass of the stone, m = 0.05 kg
During its upwards motion,
Net force acting on the stone is
$F = mg = 0.05 \text{ kg} \times 9.8 \text{ m/s}^2$
= 0.49 N, vertically downwards.

74. (c): Here, L_1 = 8 mH and L_2 = 2 mH
Induced voltage in the coil is

$$V = -L\frac{di}{dt}$$

Since $\frac{di}{dt}$ is a constant,

$\therefore \quad V \propto L$

Thus, $\frac{V_2}{V_1} = \frac{L_2}{L_1} = \frac{2}{8} = \frac{1}{4}$

Power given to the two coils is same
$\therefore \quad V_1 i_1 = V_2 i_2$

$$\frac{i_2}{i_1} = \frac{V_1}{V_2} = 4$$

Energy stored in a coil, $W = \frac{1}{2}Li^2$

$$\therefore \quad \frac{W_2}{W_1} = \left(\frac{L_2}{L_1}\right)\left(\frac{i_2}{i_1}\right)^2 = \left(\frac{2}{8}\right)(4)^2 = 4$$

75. (c): As the image is real and size of image is n times the size of object.

$\therefore$ Magnification, $m = -\frac{v}{u} = -n$
or $\quad v = un$
According to mirror equation

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \text{ or } \frac{1}{un} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1+n}{nu} = \frac{1}{f} \text{ or } \frac{nu}{1+n} = f \quad \text{or } u = \frac{(n+1)f}{n}$$

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UNIT-1 PHYSICS AND MEASUREMENT

MEASUREMENT AND PHYSICAL QUANTITY

Measurement is a process of comparing a property to a well defined and agreed-upon referent.

The well defined and agreed-upon referent is used as a standard called a **unit**.

Every measurement is a comparison of a quantity with a standard quantity that is an agreed-upon quantity of the same kind.

Those quantities which describe the laws of physics are called **physical quantities**. In order to determine physical quantity completely, we should know :

(i) Numerical value only ratio
e.g., refractive index, dielectric constant, etc.

(ii) Magnitude only scalar
e.g., mass, charge, current etc.

(iii) Magnitude and direction vector
e.g., displacement, torque, etc.

Types of Physical Quantities

Fundamental quantities : The physical quantities which can be treated independently as basic quantities, and not usually defined in terms of other physical quantities are called fundamental quantities.

e.g., length, mass, time, etc.

Only four additional fundamental quantities temperature, current, luminous intensity and amount of substance are needed to deal with all other branches of physics.

Derived quantities : The physical quantities which can be defined using fundamental quantities are called derived quantities.

e.g., speed, volume, force, etc.

Apart from fundamental and derived units we also sometimes come across practical units.

UNIT

Unit is defined as the standard of measurement of quantity.

To determine the magnitude of a physical quantity we need

(i) the unit in which it is measured

(ii) the number of times that unit is contained in the quantity.

Measurement of a physical quantity

$$= \text{numerical value } (\eta) \times \text{unit } (u).$$

e.g., measurement of length = 1 m

Here, $\eta = 1$, $u = \text{m}$

The units selected for the measurement of the base quantities are known as the base units or absolute units or fundamental units.

The units of all those quantities which can be obtained from the fundamental units are called derived units.

SYSTEM OF UNITS

A complete set of base and derived units is called as system of units.

A few common systems are given below :

- **CGS system :** This system is also called Gaussian system of units. In it length, mass and time have been taken as the fundamental quantities, and corresponding fundamental units are centimetre (cm), gram (g) and second (s) respectively.
- **FPS system :** In this system foot, pound and second are the fundamental units for the measurements of length, mass and time respectively. In this system force is a derived quantity with unit poundal.
- **MKS system :** In this system the length, mass and time have been taken as the fundamental quantities, and the corresponding fundamental units are metre, kilogram and second. The units of all other mechanical quantities like force, work, power, etc., are derived in terms of these fundamental units.
- **International system of units [SI units] :** In 1971 the International Bureau of Weights and Measures decided a system of units which is known as the international system of units. It is abbreviated as SI units from the French name Le Système Internationale d Unites and is the extended MKS system applied to the whole of physics. Now-a-days, most of the engineers and physicists use this system of units.

SUPPLEMENTARY SI UNITS

Radian (rad) : It is defined as the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.

$$\theta \text{ (in radian)} = \frac{\text{Arc}}{\text{Radius}} = \frac{l}{r}$$

Steradian (Sr) : It is the solid angle subtended at the centre of a sphere by a surface of the sphere equal in area to that of a square, having each side equal to the radius of the sphere.

$$\left(\Omega \text{ (in steradian)} = \frac{\text{Surface area}}{\text{Radius}^2}\right)$$

PRACTICAL UNITS OF LENGTH

S.No.	Practical units
1.	1 astronomical unit = 1 AU = 1.496×10^{11} m (average distance of the sun from the earth)
2.	1 light year = 1 ly = 9.46×10^{15} m (distance that light travels with velocity of 3×10^8 m s^{-1} in 1 year)
3.	1 parsec = 3.08×10^{16} m (parsec is the distance at which average radius of earth's orbit subtends an angle of 1 arc second)
4.	1 micron = 1 μm = 10^{-6} m
5.	1 angstrom = 1 Å = 10^{-10} m
6.	1 fermi = 1 fm = 10^{-15} m

PRACTICAL UNITS OF MASS

S.No.	Practical units
1.	1 tonne or 1 metric ton = 10^3 kg
2.	1 quintal = 10^2 kg
3.	1 pound = 0.4536 kg
4.	1 atomic mass unit = 1 amu or 1 u = 1.66×10^{-27} kg

PRACTICAL UNITS OF TIME

S.No.	Practical units
1.	**Solar day :** It is the time taken by earth to complete one rotation about its axis w.r.t. the sun. 1 solar day = 86400 s
2.	**Lunar month :** It is the time taken by moon to complete one revolution around the earth in its orbit. 1 lunar month = 27.3 days
3.	**Solar year :** It is the time taken by the earth to complete one revolution around the sun in its orbit. 1 solar year = $365\frac{1}{4}$ solar days.
4.	**Tropical year :** It is that year in which solar eclipse occurs.
5.	**Leap year :** It is that year in which the month of February is of 29 days.
6.	1 shake = 10^{-8} s.

ACCURACY AND PRECISION

Every physical measurement is a comparison of two similar physical quantities. To be valid, a measuring device must be compared against a widely accepted standard. Accuracy describes how much a measurement might differ from another measurement made with greater care.

The precision of a measurement is the smallest amount of the measured quantity that can reliably be distinguished. Greater precision requires a more carefully manufactured device.

DIMENSIONS OF A PHYSICAL QUANTITY

If any derived unit depends upon the r^{th} power of the fundamental unit, it is said to be of r dimensions in that fundamental unit.

Also, all physical quantities can be expressed in terms of seven fundamental quantities. These seven physical quantities are called the seven dimensions of the physical world and are denoted by brackets "[ ]".

Dimensional Formula

Dimensional formula of a derived physical quantity is an expression which of the fundamental quantities along with their powers, are required to represent that quantity.

Dimensional Equation

Dimensional equation of a physical quantity is the equation obtained by equating that quantity to its dimensional formula.

S.I. Units, definition and dimensional formulae—synopsis

S. No.	Quantity	Definition	Dimensional Formula	S.I. Unit
1.	Momentum	Mass × velocity	$[MLT^{-1}]$	kg m s^{-1}
2.	Force	Mass × acceleration	$[MLT^{-2}]$	kg m s^{-2} = newton
3.	Angle	Arc/radius	$[M^0L^0T^0]$	radian
4.	Moment of Inertia (I)	Mass × (distance)^2	$[ML^2]$	kg m^2
5.	Angular Velocity (ω)	Angle/time	$[T^{-1}]$	rad s^{-1}
6.	Angular Acceleration (α)	Angle/time^2	$[T^{-2}]$	rad s^{-2}
7.	Impulse	Force × time	$[MLT^{-1}]$	newton second
8.	Pressure	Force/area	$[ML^{-1}T^{-2}]$	pascal
9.	Work	Force × displacement	$[ML^2T^{-2}]$	joule
10.	Energy	Work	$[ML^2T^{-2}]$	joule
11.	Power	Work/time	$[ML^2T^{-3}]$	joule s^{-1} = watt
12.	Density	Mass/volume	$[ML^{-3}]$	kg m^{-3}
13.	Torque	$I \times \alpha$	$[ML^2T^{-2}]$	N m
14.	Angular Momentum	$J = I\omega$	$[ML^2T^{-1}]$	joule second
15.	Moment of Momentum	Momentum × length of arm	$[ML^2T^{-1}]$	joule second
16.	Frequency	Vibrations/second	$[T^{-1}]$	hertz
17.	Spring constant	Force/length	$[MT^{-2}]$	N m^{-1}
18.	Gravitation constant (G)	$G = \frac{Fr^2}{m^2}$	$[M^{-1}L^3T^{-2}]$	N m^2 kg^{-2}
19.	Surface Tension	Force/length	$[MT^{-2}]$	N m^{-1}
20.	Surface Energy	Energy/area	$[MT^{-2}]$	joule m^{-2}

21.	Velocity Gradient	Change in vel./dist. $= \frac{dv}{dx}$	$[T^{-1}]$	s^{-1}
22.	Coefficient of viscosity (η)	$\eta = \frac{F}{A \cdot \frac{dv}{dx}}$	$ML^{-1}T^{-1}]$	$kg\ m^{-1}s^{-1}$ or pascal second
23.	Stress	Restoring force/area	$ML^{-1}T^{-2}]$	$N\ m^{-2}$
24.	Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	$M^0L^0T^0$ or Dimensionless	No unit
25.	Modulus of Elasticity	$\frac{\text{Stress}}{\text{Strain}}$	$[ML^{-1}T^{-2}]$	$N\ m^{-2}$
26.	Poisson's Ratio	$\frac{\text{Lateral strain}}{\text{Longitudinal strain}}$	$M^0L^0T^0$ or Dimensionless	No unit
27.	Couple	Force × distance	$[ML^2T^{-2}]$	N m
28.	Angular Impulse	Couple × time	$[ML^2T^{-1}]$	N m s

Heat

S. No.	Quantity	Definition	Dimensional Formula	S.I. Unit
1.	Heat (Q)	Energy	$[ML^2T^{-2}]$	joule 4.2 joule = 1 calorie
2.	Mechanical equivalent of Heat	$J = \frac{W}{H}$	$[M^0L^0T^0]$	joule $(\text{calorie})^{-1}$
3.	Specific heat	$s = \frac{Q}{m \times \theta}$	$[L^2T^{-2}\theta^{-1}]$	joule $kg^{-1}\ K^{-1}$
4.	Absolute Temperature	—	θ	kelvin = K
5.	Thermal Capacity	Mass × specific heat	$[ML^2T^{-2}\theta^{-1}]$	joule K^{-1}
6.	Molar sp. heat	Specific heat × molecular wt.	$[ML^2T^{-2}\theta^{-1}mol^{-1}]$	joule $K^{-1}\ (mol)^{-1}$
7.	Latent Heat	Heat/mass	$[L^2T^{-2}]$	joule kg^{-1}
8.	Entropy	Heat/temperature	$[ML^2T^{-2}\theta^{-1}]$	joule K^{-1}
9.	Solar Constant	$\frac{\text{Energy}}{\text{Time} \times \text{area}}$	$[MT^{-3}]$	watt m^{-2}
10.	Gas constant (R)	$R = \frac{PV}{nT}$	$[ML^2T^{-2}\theta^{-1}mol^{-1}]$	joule $mol^{-1}\ K^{-1}$
11.	Stefan's constant (σ)	$\frac{\text{Energy}}{\text{Area} \times \text{time} \times (\text{temperature})^4}$	$[MT^{-3}\theta^{-4}]$	watt $m^{-2}\ K^{-4}$
12.	Wien's constant (b)	$b = \lambda_m \times$ temperature	L θ	mK
13.	Boltzmann constant	$\frac{\text{Energy}}{\text{Temperature}}$	$[ML^2T^{-2}\theta^{-1}]$	joule K^{-1}

14.	Planck's constant (h)	$h = \frac{\text{Energy}}{\text{Frequency}}$	[ML^2T^{-1}]	joule second
15.	Van der Waal constants (a) and (b)	(a) a = Pressure × (Volume)^2 (b) b = Volume correction	[$ML^5T^{-2}L^3$]	N m^4 m^3
16.	Coefficient of expansion (Linear, superficial, volume, pressure)	Change in quantity per unit quantity per unit temperature	θ^{-1}	K^{-1}
17.	Temperature gradient	$d\theta/dx$	$L^{-1}\theta$	K m^{-1}
18.	Coeff. of thermal conductivity	$\frac{\text{Heat current} \times \text{length}}{\text{Cross-sect.area} \times \text{temp.diff.}}$	[$MLT^{-3}\theta^{-1}$]	$\frac{\text{joule}}{\text{msK}} = \frac{\text{watt}}{\text{mK}}$
19.	Thermometric conductivity or Thermal diffusivity	Ratio of coefficient of thermal conductivity and thermal capacity per unit volume	[L^2T^{-1}]	m^2s^{-1}
20.	Thermal Resistance	$\frac{\text{Distance}}{\text{Thermal conductivity} \times \text{area}}$	[$M^{-1}L^{-2}T^3\theta$]	$\frac{\text{K s}}{\text{joule}}$
21.	Emissive Power	$\frac{\text{Heat}}{(\text{Area} \times \text{time})}$	[MT^{-3}]	$\frac{\text{joule}}{m^2 s} = \frac{\text{watt}}{m^2}$
22.	Rate of cooling	$\frac{\text{Heat lost}}{\text{Time}}$	[ML^2T^{-3}]	$\frac{\text{joule}}{\text{sec}}$
23.	Molar volume	$\frac{\text{Volume}}{\text{No. of moles}}$	[L^3 mol^{-1}]	$\frac{m^3}{\text{mol}}$

Waves—Light and Sound

S. No.	Quantity	Definition	Dimensional Formula	S.I. Unit
1.	Luminous flux	$\frac{\text{Light energy}}{\text{second}}$	ML^2T^{-3} or cd	watt = lumen
2.	Luminous intensity	$\frac{\text{Luminous flux}}{\text{Solid angle}}$	$\frac{\text{cd}}{\text{sr}} = \text{cd}\,\text{sr}^{-1}$	$\frac{\text{lumen}}{\text{steradian}}$
3.	Density of wave	$\frac{\text{Energy}}{\text{Time} \times \text{area}}$	MT^{-3}	$\frac{\text{watt}}{m^2}$
4.	Energy density	$\frac{\text{Energy}}{\text{Volume}}$	$ML^{-1}T^{-2}$	joule m^{-3}
5.	Intensity of wave	$\frac{\text{Energy}}{\text{Area} \times \text{time}}$	MT^{-3}	watt m^{-2}

Electricity				
S. No.	**Quantity**	**Definition**	**Dimensional Formula**	**S.I. Unit**
1.	Charge	Current × time	[TA]	coulomb (C)
2.	Electric potential (V)	Work/charge	$[ML^2T^{-3}A^{-1}]$	volt = joule C^{-1}
3.	Electric intensity	Force/charge	$[MLT^{-3}A^{-1}]$	$\frac{\text{newton}}{\text{coulomb}} = NC^{-1}$
4.	Capacitance	Charge/potential	$[M^{-1}L^{-2}T^4A^2]$	farad
5.	Electric dipole moment	Charge × distance	[LTA]	Cm
6.	Electrical permittivity (ε_0)	$\frac{q_1 q_2}{4\pi Fr^2} = \varepsilon_0$	$[M^{-1}L^{-3}T^4A^2]$	$C^2N^{-1}m^{-2}$
7.	Dielectric constant or relative permittivity	$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$	$[M^0L^0T^0]$	Unitless
8.	Resistance (R)	$\frac{\text{Potential difference}}{\text{Current}}$	$[ML^2T^{-3}A^{-2}]$	ohm (Ω)
9.	Conductance	$(\text{Resistance})^{-1}$	$[M^{-1}L^{-2}T^3A^2]$	Siemen = $(\text{ohm})^{-1}$
10.	Resistivity or Specific resistance	$\frac{\text{Area} \times \text{resistance}}{\text{Length}}$	$[ML^3T^{-3}A^{-2}]$	ohm metre
11.	Conductivity	$(\text{Resistivity})^{-1}$	$[M^{-1}L^{-3}T^3A^2]$	ohm^{-1} metre^{-1}
12.	Electric flux	Electric intensity × area	$[ML^3T^{-3}A^{-1}]$	$N\ C^{-1}m^2$
13.	Specific charge	Charge/mass	$[M^{-1}TA]$	coulomb kg^{-1}

Magnetism				
S. No.	**Quantity**	**Definition**	**Dimensional Formula**	**S.I. Unit**
1.	Magnetic moment	Current × area	$[L^2A]$	ampere m^2
2.	Pole strength	Magnetic moment/length	[LA]	ampere metre
3.	Intensity of magnetization (I)	$\frac{\text{Magnetic moment}}{\text{volume}}$	$[L^{-1}A]$	Am^{-1}
4.	Coefficient of self inductance	$\frac{emf}{dI/dt}$	$[ML^2T^{-2}A^{-2}]$	henry
5.	Coefficient of mutual inductance	$\frac{emf \text{ in secondary}}{dI_p/dt}$	$[ML^2T^{-2}A^{-2}]$	henry
6.	Magnetic field or magnetic intensity (H)	$\frac{Idl\sin\theta}{4\pi r^2}$	$[L^{-1}A]$	ampere m^{-1}

7.	Magnetic induction or magnetic flux density (B)	$B = \dfrac{F}{Il\sin\theta}$	$[MT^{-2}A^{-1}]$	tesla or $\dfrac{\text{weber}}{\text{m}^2}$
8.	Magnetic flux (ϕ)	$\phi = B \times \text{area}$	$[ML^2T^{-2}A^{-1}]$	weber (Wb)
9.	Magnetic permeability (μ)	$\dfrac{4\pi r^2 (dB)}{Idl\sin\theta}$	$[MLT^{-2}A^{-2}]$	$\dfrac{\text{weber}}{\text{ampere} \times \text{metre}}$
10.	Relative permeability	$\mu_r = \dfrac{\mu}{\mu_0}$	$[M^0L^0T^0]$	No unit
11.	Magnetic susceptibility	$\chi = \dfrac{I}{H}$	$[M^0L^0T^0]$	No unit

PRINCIPLE OF HOMOGENEITY OF DIMENSIONS

The powers of mass, length and time in each term on one side of a dimensional equation must be equal to their respective powers in each form on the other side of the dimensional equation.

Uses of Dimensional Equations

(i) To convert a physical quantity from one system of units to another.

Let a physical quantity has dimensional formula $M^aL^bT^c$. Let n_1 and n_2 be its numerical values when the units are v_1 and v_2, then

$$n_1v_1 = n_2v_2$$

$$n_1[M_1^aL_1^bT_1^c] = n_2[M_2^aL_2^bT_2^c]$$

$$\therefore \quad n_2 = n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

(ii) To verify whether an equation connecting physical quantity is correct or not.

(iii) To check the correctness of a given physical relation.

(iv) To derive a relationship between different physical quantities.

Note :

(i) Dimensional analysis treats dimensions as algebraic quantities.

(ii) In dimensional analysis, we add or subtract only the magnitudes of the same physical quantity and the resultant is a magnitude of the same physical quantity.

ERROR IN A MEASUREMENT

It is the difference between the measured value and the true value of a physical quantity. It gives an indication of the limits within which the true value may lie.

THERE ARE THREE TYPES OF ERRORS :

- Systematic errors
- Random errors
- Gross errors

Systematic Errors

Errors whose causes are known are called systematic errors. These errors can be minimized by improving experimental techniques, selecting better instruments and removing personal bias as far as possible.

Systematic errors are of various types :

(i) **Errors due to external factors :** These errors are due to fluctuation in atmospheric conditions like temperature, pressure, humidity, etc.

(ii) **Errors due to imperfection :** These are introduced due to negligence of facts. For example, error in weighing of a body arising out of buoyancy, which is usually ignored.

(iii) **Instrumental errors :** These errors are introduced due to improper designing and manufacturing defects of instrument. Often there may be zero error. For example, a meter scale may be worn off at the end of zero mark. The instrumental errors can be reduced by using more accurate instruments and applying zero correction, when required.

(iv) **Personal errors :** These errors are introduced due to lack of proper care of the observer. For example, lack of proper setting of the apparatus, recording the reading without applying proper precautions and so on.

Random Errors

Error occur randomly. The causes of such errors are not known precisely. Hence, it is not possible to eliminate the random errors. For example, same person repeating the same experiment may get different readings each time.

These errors can be minimized by repeating the experiment and taking the arithmetic mean of all the observations. The mean value should be close to the accurate value.

Gross Errors

These errors arise on the account of shear carelessness of the observer. For example :

Reading an instrument without setting it properly.

Recording the observations wrongly without caring for the sources of errors.

Using wrong values of the observations in calculations. These errors can be minimized only if observer is sincere and mentally alert.

Least count error : The smallest value that can be measured by the measuring instrument is known as its least count. All the readings or measured values are good only upto this value.

The least count error is the error associated with the resolution of the instrument.

Least count error belongs to the category of random errors but within a limited size; it occurs with both systematic and random errors.

Using instruments of higher precision, improving experimental techniques, etc., we can reduce the least count error.

Elimination of random error : To eliminate random error, a large number of readings are taken and their arithmetic mean is taken as the true value.

$$\bar{a} = \frac{a_1 + a_2 + a_3 + \ldots + a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i$$

ABSOLUTE, MEAN ABSOLUTE RELATIVE AND PERCENTAGE ERRORS

Absolute error : The magnitude of the difference between the true value and the measured value is called absolute error. Such errors are given by

$$\Delta a_1 = \bar{a} - a_1 \;;\; \Delta a_2 = \bar{a} - a_2$$

$$\Delta a_3 = \bar{a} - a_3 \;;\; \Delta a_n = \bar{a} - a_n$$

Mean absolute error : The arithematic mean of the positive magnitudes of all the absolute errors called mean absolute error.

$$\Delta\bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n}|\Delta a_i|$$

Relative error : It is the ratio of the mean absolute error to the mean value.

Percentage error : The relative error expressed in percent is called the percentage error.

$$\text{Percentage error} = \frac{\Delta\bar{a}}{a} \times 100\%$$

ERROR COMBINATION IN A SUM OR A DIFFERENCE

When two quantities are added or subtracted, the absolute error in the final result is the sum of the absolute errors associated with the individual quantities.

if, $Z = A \pm B$

$\Rightarrow$ $\Delta Z = \Delta A + \Delta B$

ERROR COMBINATION IN A PRODUCT OR A QUOTIENT

When two quantities are multiplied or divided, the fractional error in the final result is the sum of the fractional errors of the two quantities.

$$\frac{\Delta Z}{Z} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Error Due to the Power of a Measured Quantity

The fractional error in the n^{th} power of a quantity is equal to n times the fractional error in the quantity itself.

If $Z = A^n$, then $\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$

General rule : If $Z = \frac{A^p B^q}{C^r}$, then the maximum fractional or relative error in Z will be

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

$$\%\text{ error in } Z = \frac{\Delta Z}{Z} \times 100$$

$$= p\frac{\Delta A}{A} \times 100 + q\frac{\Delta B}{B} \times 100 + r\frac{\Delta C}{C} \times 100$$

Propagation or combination of errors : It is summarised in the table given below :

Operation	Formula Z	Absolute error ΔZ	Relative error $\Delta Z/Z$	Percentage error $\Delta Z/Z \times 100$
Sum	$A + B$	$\Delta A + \Delta B$	$\dfrac{\Delta A + \Delta B}{A + B}$	$\dfrac{\Delta A + \Delta B}{A + B} \times 100$
Difference	$A - B$	$\Delta A + \Delta B$	$\dfrac{\Delta A + \Delta B}{A - B}$	$\dfrac{\Delta A + \Delta B}{A - B} \times 100$
Multiplication	$A \times B$	$A\Delta B + B\Delta A$	$\dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$	$\left(\dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}\right) \times 100$
Division	$\dfrac{A}{B}$	$\dfrac{B\Delta A + A\Delta B}{B^2}$	$\dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$	$\left(\dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}\right) \times 100$
Power	A^n	$nA^{n-1}\Delta A$	$n\dfrac{\Delta A}{A}$	$n\dfrac{\Delta A}{A} \times 100$
Root	$A^{1/n}$	$\dfrac{1}{n} A^{\frac{1}{n-1}} \Delta A$	$\dfrac{1}{n}\dfrac{\Delta A}{A}$	$\dfrac{1}{n}\dfrac{\Delta A}{A} \times 100$

SIGNIFICANT FIGURES

The number of digits in the measured value which include certain digits plus one uncertain digit (doubtful digit) are called significant figures.

RULES FOR COUNTING THE SIGNIFICANT FIGURES

Rule I - All non-zero digits are significant.

Rule II - All zeros occurring between the non-zero digits are significant. *i.e.* 230089 contains six significant figures.

Rule III - All zeros to the left of non-zero digit in a number with or without decimal point are not significant. *i.e.* 0.0023 contains two significant figures.

Rule IV - All zeros to the right of non-zero digits in a number without decimal point are not significant. *e.g.* 23000 contains two significant figures.

The zeros to the right of non-zero digits (trailing zeros) in a number with a decimal point are significant. *e.g.* 0.2300 contains four significant figures.

Remember : Forget zeros on left but do not forget zeros on the right. Also please note that in scientific notation 23000 should be written as 23.000×10^3. Thus 23 thousands is written as 23×10^3 and it contains two significant figures. Similarly, 230 hundreds is written as 230×10^2 and it contains three significant digits. Thus power (or exponent) of 10 is irrelevant in finding the significant figures in scientific notation. However, all the trailing zeros appearing in the base number in it are significant.

The change of units only changes the order of exponent but not the number of significant figures. *e.g.* 1.40 m = 1.40×10^2 cm both have three significant figures.

ROUNDING OFF

Rule 1 - If the digit to be dropped is less than 5, then the preceding digit is left unchanged. *e.g.* 8.22 is rounded off to 8.2.

Rule 2 - If the digit to be dropped is more than 5, then the preceding digit is raised by one. *e.g.* 6.87 is rounded off to 6.9.

Rule 3: If the digit to be dropped is 5 followed by digit other than zero, then the preceeding digit is raised by one. *e.g.* 7.851 is rounded off to 7.9.

Rule 4 - If the digit to be dropped is 5 or 5 followed by zero, then preceding digit is left unchanged, if it is even. *e.g.* 5.250 rounded off to 5.2.

Rule 5 - If the digit to be dropped is 5 or 5 followed by zero, then the preceding digit is raised by one, if it is odd. *e.g.* 3.750 is rounded off to 3.8.

Accuracy is the extent to which a reported measurement approaches the true value of the quantity measured.

QUESTIONS FOR PRACTICE

1. In a vernier callipers, N divisions of vernier scale coincide with $(N - 1)$ division of main scale (in which one division represents 1 mm). The vernier constant is (cm)
(a) N (b) $N - 1$
(c) $\frac{1}{N-1}$ (d) $\frac{1}{10N}$

2. The side of a cubical block when measured with a vernier callipers is 2.50 cm. The vernier constant is 0.01 cm. The maximum possible error in the area of the side of the block is
(a) ± 0.01 cm^2 (b) ± 0.02 cm^2
(c) ± 0.05 cm^2 (d) ± 0.10 cm^2

3. A physical quantity is given by $X = M^aL^bT^c$. The percentage error in measurement of M, L and T are α, β and γ respectively. Then, the maximum % error in the quantity X is
(a) $a\alpha + b\beta + c\gamma$ (b) $a\alpha + b\beta - c\gamma$
(c) $\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}$ (d) none of these.

4. If η denotes coefficient of viscosity and G denotes gravitational constant then $G \times \eta$ yields the dimensions
(a) $M^2L^0T^{-3}$ (b) MLT^{-3}
(c) ML^2T^{-2} (d) $M^0L^2T^{-3}$

5. A certain body weighs 22.42 g and has a measured volume of 4.7cc. The possible error in the measurement of mass and volume are 0.01 g and 0.1 cc. Then maximum error in the densiy will be
(a) 22% (b) 2%
(c) 0.2% (d) 0.02%.

6. In the formula, $X = 3YZ^2$; X has dimensions of capacitance and Z has dimensions of magnetic induction. The dimensions of Y are
(a) $[M^{-3}L^{-2}T^{-2}A^4]$ (b) $[ML^{-2}T^2A^2]$
(c) $[M^{-3}L^{-2}A^4T^4]$ (d) $[M^{-3}L^{-2}T^8A^4]$

7. If the energy, $E = G^ph^qc^r$, where G is the universal gravitational constant, h is the Planck's constant and c is the velocity of light, then the values of p, q and r are, respectively
(a) $-\frac{1}{2}, \frac{1}{2}$ and $\frac{5}{2}$ (b) $\frac{1}{2}, -\frac{1}{2}$ and $-\frac{5}{2}$
(c) $-\frac{1}{2}, \frac{1}{2}$ and $\frac{3}{2}$ (d) $\frac{1}{2}, -\frac{1}{2}$ and $-\frac{3}{2}$

8. Three measurements are made as 18.425 cm, 7.21 cm and 5.0 cm. The sum of measurements upto correct number of significant figures is
(a) 30.635 cm (b) 30.64 cm
(c) 30.63 cm (d) 30.6 cm

9. In an experiment of simple pendulum, the errors in the measurement of length of the pendulum (L) and time period (T) are 3% and 2% respectively. The maximum percentage error in the value of $\frac{L}{T^2}$ is
(a) 5% (b) 7% (c) 8% (d) 1%

10. The moment of inertia of a body rotating about a given axis is 12.0 kg m^2 in the SI system. What is the value of the moment of inertia in a system of units in which the unit of length is 5 cm and the unit of mass is 10 g?
(a) 2.4×10^3 (b) 6.0×10^3
(c) 5.4×10^5 (d) 4.8×10^5

11. The dimension of angular momentum is
(a) $M^0L^1T^{-1}$ (b) $M^1L^2T^{-2}$
(c) $M^1L^2T^{-1}$ (d) $M^2L^1T^{-2}$

12. In a slide callipers, $(m + 1)$ number of vernier divisions is equal to m number of smallest main scale divisions. If d unit is the magnitude of the smallest main scale division, then the magnitude of the vernier constant is
(a) $\frac{d}{(m+1)}$ unit (b) $\frac{d}{m}$ unit
(c) $\frac{md}{(m+1)}$ unit (d) $\frac{(m+1)d}{m}$ unit

13. The equation of state of a gas is given by $\left(P+\frac{a}{V^3}\right)(V-b^2)=cT$, where P, V, T are pressure, volume and temperature respectively, and a, b, c are constants. The dimensions of a and b are respectively
(a) ML^8T^{-2} and $L^{3/2}$ (b) ML^5T^{-2} and L^3
(c) ML^5T^{-2} and L^6 (d) ML^6T^{-2} and $L^{3/2}$

14. The time dependence of a physical quantity p is given by $p = p_0 \exp(-\alpha t^2)$, where α is a constant and t is the time. The constant α
(a) is dimensionless
(b) has dimensions $[T^{-2}]$
(c) has dimensions $[T^2]$
(d) has dimensions of p

15. An important milestone in the evolution of the universe just after the Big Bang is the Planck time t_p, the value of which depends on three fundamental constants-speed of light in vacuum c, Gravitational constant G and Planck's constant h. Then, $t_p \propto$
(a) Ghc^5 (b) $\frac{c^5}{Gh}$
(c) $\frac{Gh}{c^5}$ (d) $\left(\frac{Gh}{c^5}\right)^{1/2}$

16. The mass of a body is 20.0 g and its volume is 10.0 cm^3. If possible maximum errors in the measurement of mass of body and volume of body are 0.001 g and 0.01 cm^3 respectively, then the maximum error in the value of density is
(a) 0.001 g cm^{-3} (b) 0.010 g cm^{-3}
(c) 0.10 g cm^{-3} (d) none of these

17. If 3.8×10^{-6} is added to 4.2×10^{-5} giving due regard to significant figures, then the result will be
(a) 4.58×10^{-5} (b) 4.6×10^{-5}
(c) 4.5×10^{-5} (d) 4.7×10^{-5}

18. When one metre, one kg and one minute are taken as fundamental units, the magnitude of a force is 36 units. What is the value of this force in CGS system?
(a) 10^3 dyne (b) 10^5 dyne
(c) 10^6 dyne (d) 10^7 dyne

19. The dimensions of σb^4 (σ = Stefan's constant and b = Wein's constant) are
(a) $[M^0L^0T^0]$ (b) $[ML^4T^{-3}]$
(c) $[ML^{-2}T]$ (d) $[ML^6T^{-3}]$

20. In an experiment to measure the height of a bridge by dropping stone into water underneath, if the error in measurement of time is 0.1 s at the end of 2 s, then the error in estimation of height of bridge will be
(a) 0.49 m (b) 0.98 m
(c) 1.96 m (d) 2.11 m

21. If E, m, l and G denote energy, mass, angular momentum and gravitational constant respectively, the quantity (El^2/m^5G^2) has the dimensions of
(a) angle (b) length
(c) mass (d) time

22. In the following equation, x, t and F represent respectively, displacement, time and force :

$$F = a + bt + \frac{1}{c+dx} + A\sin(\omega t + \phi).$$

The dimensional formula for $A \cdot d$ is
(a) T^{-1} (b) L^{-1} (c) M^{-1} (d) TL^{-1}

23. The least count of the metre rod is 0.1 cm. What is the permissible error in the length of the rod measured with it ?
(a) ± 0.2 cm (b) ± 0.1 cm
(c) ± 0.05 cm (d) ± 0.01 cm

24. A student performs an experiment for determination of $g\left(=\frac{4\pi^2 l}{T^2}\right)$, l = 1 m, and he commits an error of Δl. For T he takes the time of n oscillations with the stop watch of least count ΔT and he commits a human error of 0.1 sec. For which of the following data, the measurement of g will be most accurate?

	Δl	ΔT	n	amplitude of oscillation
(a)	5 mm	0.2 sec	10	5 mm
(b)	5 mm	0.2 sec	20	5 mm
(c)	5 mm	0.1 sec	20	1 mm
(d)	1 mm	0.1 sec	50	1 mm

25. Which two of the following five physical parameters have the same dimensions ?

1. energy density 2. refractive index
3. dielectric constant 4. Young's modulus
5. magnetic field

(a) 1 and 4 (b) 1 and 5
(c) 2 and 4 (d) 3 and 5

26. In a new system of units, unit of mass is 10 kg, unit of length is 1 km and unit of time is 1 minute. The value of 1 joule in this new hypothetical system is

(a) 3.6×10^{-4} new units
(b) 6×10^{7} new units
(c) 10^{11} new units
(d) 1.67×10^{4} new units

27. The ratio of the dimensions of Planck constant and that of moment of inertia is the dimensions of

(a) time
(b) frequency
(c) angular momentum
(d) velocity

28. Which of the following has the highest number of significant figures?

(a) 1.64×10^{20} kg (b) 0.006 m^2
(c) 7.2180 J (d) 5.045 J

29. If E denotes the electric field and ε_0 is the permittivity of free space, the dimensional formula of $\frac{1}{2}\varepsilon_0 E^2$ is

(a) $[ML^2T^{-2}]$ (b) $[ML^2T^{-1}]$
(c) $[ML^{-1}T^{-2}]$ (d) $[MLT^{-2}]$

30. The dimensions of resistance × capacitance are same as that of

(a) current (b) energy
(c) frequency (d) time-period.

SOLUTIONS

1. (d) : N VSD = $(N-1)$ MSD

$$1\ \text{VSD} = \frac{N-1}{N}\ \text{MSD}$$

V.C. = 1 MSD – 1 VSD

$$= 1\ \text{MSD} - \left(\frac{N-1}{N}\right)\text{MSD} = \frac{1}{N}\ \text{MSD}$$

$$= \frac{1}{N}\ \text{mm} = \frac{1}{10N}\ \text{cm}$$

2. (c) : Here $l = 2.50$ cm and $\Delta l = 0.01$ cm

Since $A = l^2 = (2.50\ \text{cm})^2$

$$\therefore \quad \frac{\Delta A}{A} = 2 \cdot \frac{\Delta l}{l};\ \frac{\Delta A}{A} = 2 \times \frac{0.01\ \text{cm}}{2.50\ \text{cm}}$$

$$\therefore \quad \Delta A = \frac{2 \times 0.01\ \text{cm}}{2.50\ \text{cm}} \times (2.50\ \text{cm})^2$$

$$= 2 \times 0.01 \times 2.50\ \text{cm}^2$$

$$= 0.01 \times 5\ \text{cm}^2$$

or $\Delta A = \pm 0.05\ \text{cm}^2$

3. (a) : $X = M^a L^b T^c$

Percentage error in X

$$\frac{\Delta X}{X} \times 100 = a\frac{\Delta M}{M} \times 100 + b\frac{\Delta L}{L} \times 100 + c\frac{\Delta T}{T} \times 100$$

As given,

$$\frac{\Delta M}{M} \times 100 = \alpha,\ \frac{\Delta L}{L} \times 100 = \beta,\ \frac{\Delta T}{T} \times 100 = \gamma$$

$\therefore$ Percentage error in $X = a\alpha + b\beta + c\gamma$.

4. (d) : $[G \times \eta] = [M^{-1}L^3T^{-2}]\ [ML^{-1}T^{-1}]$
$= [M^0L^2T^{-3}]$

5. (b) : Density $\rho = \dfrac{\text{mass } m}{\text{volume } V}$ ...(i)

Take logarithm on the both sides of eqn. (i), we get

$\ln\rho = \ln m - \ln V$ ...(ii)

Differentiate eqn. (ii), on both sides, we get

$$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} - \frac{\Delta V}{V}$$

Errors are always added for maximum error.

$\therefore$ Maximum error in the density ρ will be

$$=\left[\frac{\Delta m}{m}+\frac{\Delta V}{V}\right]\times 100\,\%$$

$$=\left[\frac{0.01}{22.42}+\frac{0.1}{4.7}\right]\times 100\,\% = 2\%$$

6. **(d) :** As $q = CV$,

$$\Rightarrow \ C=\frac{q}{V}=\frac{q^2}{W} \quad \left[\text{as } V=\frac{W}{q}\right]$$

$$[X]\rightarrow[C]=\left[\frac{\mathrm{A^2T^2}}{\mathrm{ML^2T^{-2}}}\right]=[\mathrm{M^{-1}L^{-2}T^4A^2}]$$

$$F = BIl\sin\theta,\ [B]=\left[\frac{F}{Il}\right]$$

$$[Z]\rightarrow[B]=\left[\frac{\mathrm{MLT^{-2}}}{\mathrm{AL}}\right]=[\mathrm{ML^0T^{-2}A^{-1}}]$$

Given: $X = 3YZ^2 \ \Rightarrow\ Y = X/(3Z^2)$

$$\text{or}\quad [Y]=\frac{[X]}{[Z^2]}=\frac{[\mathrm{M^{-1}L^{-2}T^4A^2}]}{[\mathrm{ML^0T^{-2}A^{-1}}]^2}$$

$$=[\mathrm{M^{-3}L^{-2}T^8A^4}]$$

7. **(a) :** $E = G^p h^q c^r$ ...(i)

$$[\mathrm{M^1L^2T^{-2}}] = [\mathrm{M^{-1}L^3T^{-2}}]^p[\mathrm{ML^2T^{-1}}]^q[\mathrm{LT^{-1}}]^r$$

$$= [\mathrm{M^{-p+q}\,L^{3p+2q+r}\,T^{-2p-q-r}}]$$

Applying principle of homogeneity of dimensions, we get

$-p + q = 1$ ...(ii)

$3p + 2q + r = 2$ ...(iii)

$-2p - q - r = -2$ ...(iv)

Adding (iii) and (iv) we get, $p + q = 0$ ...(v)

Adding (ii) and (v) we get, $2q = 1, q=\frac{1}{2}$

From (ii) $p = q-1=\frac{1}{2}-1=-\frac{1}{2}$

Put in (iii), $-\frac{3}{2}+1+r=2, r=\frac{5}{2}$

8. **(d) :** Sum of the measurements is correct only upto one place of decimal corresponding to the number with least decimal places.

$18.425 + 7.21 + 5.0 = 30.635 = 30.6$ cm

9. **(b) :** Time period of simple pendulum is

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Squaring both sides, we get

$$\therefore\ T^2 = 4\pi^2\frac{L}{g} \qquad \text{...(i)}$$

$$\text{or}\quad g = 4\pi^2\frac{L}{T^2}$$

The maximum percentage error in g is

$$\frac{\Delta g}{g}\times 100=\frac{\Delta L}{L}\times 100+2\left(\frac{\Delta T}{T}\right)\times 100$$

$$= 3\% + 2 \times 2\% = 7\%$$

From (i), we get

$$\frac{L}{T^2}=\frac{g}{4\pi^2}$$

The maximum percentage error in $\frac{L}{T^2}$ is

$$\frac{\Delta\left(\frac{L}{T^2}\right)}{\frac{L}{T^2}}\times 100=\frac{\Delta g}{g}\times 100 = 7\%$$

10. **(d) :** $n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$ ...(i)

Dimensional formula of moment of inertia

$= [\mathrm{ML^2T^0}]$

$\therefore$ $a = 1, b = 2, c = 0$

Here, $n_1 = 12.0, M_1 = 1$ kg, $M_2 = 10$ g

$L_1 = 1$ m, $L_2 = 5$ cm, $T_1 = 1$ s, $T_2 = 1$ s

$$n_2 = 12.0\left(\frac{1\text{ kg}}{10\text{ g}}\right)^1\left(\frac{1\text{ m}}{5\text{ cm}}\right)^2\left(\frac{1\text{ s}}{1\text{ s}}\right)^0 \quad \text{(Using (i))}$$

$$=12\times\left(\frac{1000\text{ g}}{10\text{ g}}\right)^1\left(\frac{100\text{ cm}}{5\text{ cm}}\right)^2\times 1$$

$$= 12 \times 100 \times 400 = 4.8 \times 10^5$$

11. **(c) :** Angular momentum = Moment of inertia × angular velocity

[Angular momentum] = $[\mathrm{M^1L^2}][\mathrm{T^{-1}}] = [\mathrm{M^1L^2T^{-1}}]$

12. **(a) :** $(m + 1)$ V.S.D. = m M.S.D.

$$1\text{ V.S.D.}=\frac{m}{m+1}\text{ M.S.D.}$$

Vernier constant = 1 M.S.D. – 1 V.S.D.

$$=1\text{ M.S.D.}-\left(\frac{m}{m+1}\right)\text{M.S.D.}$$

$$=\frac{1}{(m+1)}\text{ M.S.D.}=\frac{d}{m+1}\text{ unit}$$

13. (a) : Given, $\left(P+\frac{a}{V^3}\right)(V-b^2)=cT$

Dimensions of $[a]$ = dimensions of $[PV^3]$

$$=\left[\frac{F}{A}V^3\right] \qquad \left(\because P=\frac{F}{A}\right)$$

$$=\frac{[MLT^{-2}]}{[L^2]}\times[L^3]^3=[ML^8T^{-2}]$$

Now, dimensions of $[b]^2$ = dimensions of $[V]$

$[b]=[L^3]^{1/2}$ or $[b]=[L^{3/2}]$

14. (b) : Given : $p=p_0e^{-\alpha t^2}$

αt^2 is dimensionless

$$\therefore\quad [\alpha]=\frac{1}{[t^2]}=\frac{1}{[T^2]}=[T^{-2}]$$

15. (d) : $[Gh]=[M^{-1}L^3T^{-2}][ML^2T^{-1}]=[M^0L^5T^{-3}]$

$[c]=[LT^{-1}]$

$$\therefore\quad \left[\frac{Gh}{c^5}\right]^{1/2}=[T]$$

$$\text{Hence, } t_p\propto\left(\frac{Gh}{c^5}\right)^{1/2} \qquad (\because\ [t_p]=[T])$$

16. (d) : Maximum error in measuring mass is 0.001 g, and maximum error in measuring volume is 0.01 cm^3.

$$\frac{\Delta\rho}{\rho}=\frac{\Delta M}{M}+\frac{\Delta V}{V}=\frac{0.001}{20.0}+\frac{0.01}{10.0}$$

$$=(5\times10^{-5})+(1\times10^{-3})=1.05\times10^{-3}$$

$\Delta\rho=(1.05\times10^{-3})\times\rho$

$$=1.05\times10^{-3}\times\frac{20.0}{10.0}=0.002\text{ g cm}^{-3}$$

17. (b) : We will use the general rule of addition by making the powers same.

i.e., we add 3.8×10^{-6} and 42×10^{-6}, we get

$45.8\times10^{-6}=4.58\times10^{-5}$.

As least number of significant figures in given values are 2, so we round off the result to 4.6×10^{-5}.

18. (a) : $n_2=n_1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$

$M_1=1$ kg	$M_2=1$ g
$L_1=1$ m	$L_2=1$ cm
$T_1=1$ min	$T_2=1$ s
$n_1=36$	$n_2=?$

The dimensional formula of force is $[MLT^{-2}]$.

$\therefore\quad a=1, b=1, c=-2$

$$n_2=36\left[\frac{1\text{ kg}}{1\text{ g}}\right]^1\left[\frac{1\text{ m}}{1\text{ cm}}\right]^1\left[\frac{1\text{ min}}{1\text{ s}}\right]^{-2}$$

$$=36\left[\frac{1000\text{ g}}{\text{g}}\right]^1\left[\frac{100\text{ cm}}{1\text{ cm}}\right]^1\left[\frac{60\text{ s}}{1\text{ s}}\right]^{-2}$$

$$=\frac{36\times10^3\times10^2}{3600}=10^3$$

Hence, in the CGS system of units the value of given force is 10^3 dyne.

19. (b) : Wein's displacement law

$\lambda_mT=b$ or $b^4=\lambda_m^4T^4$ ...(i)

From Stefan's law,

$$\sigma=\frac{E}{AtT^4} \qquad ...(ii)$$

From (i) and (ii),

$$\sigma b^4=\frac{E\lambda_m^4T^4}{AtT^4}$$

$$[\sigma b^4]=\frac{[ML^2T^{-2}][L]^4}{[L^2][T]}=[ML^4T^{-3}]$$

20. (c) : $h=ut+\frac{1}{2}gt^2$

$$h=0+\frac{1}{2}\times9.8\,(2)^2=19.6\text{ m} \qquad (\because\ u=0)$$

$$\frac{\Delta h}{h}=2\left(\frac{\Delta t}{t}\right)=2\left(\frac{0.1}{2}\right)=\frac{1}{10}$$

$$\Delta h=\frac{h}{10}=\frac{19.6}{10}=1.96\text{ m}$$

21. (a) : $[E] = [ML^2T^{-2}]$, $[m] = [M]$, $[l] = [ML^2T^{-1}]$, $[G]=[M^{-1}L^3T^{-2}]$

$$\therefore \left[\frac{El^2}{m^5G^2}\right]=\frac{[ML^2T^{-2}]\cdot[M^2L^4T^{-2}]}{[M^5]\cdot[M^{-2}L^6T^{-4}]}$$

$$= [M^0L^0T^0]$$

As angle has no dimension, this has the same dimension as the angle.

22. (b) : $F = a + bt + \dfrac{1}{c+dx} + A\sin(\omega t + \phi)$

As $\sin(\omega t + \phi)$ is dimensionless, therefore A has dimensions of force.

$\therefore \quad [A] = [F] = [MLT^{-2}]$

As each term on RHS represents force

$$\therefore \left[\frac{1}{c+dx}\right]=[F]$$

$$\left[\frac{1}{c}\right]=[F]$$

$$\therefore \quad [c]=\frac{1}{[F]}=\frac{1}{[MLT^{-2}]}=[M^{-1}L^{-1}T^2]$$

As c is added to dx, therefore dimensions of c is same that of dx.

$\therefore \quad [dx] = [c]$

$$\text{or} \quad [d]=\frac{[c]}{[x]}=\frac{[M^{-1}L^{-1}T^{-2}]}{[L]}=[M^{-1}L^{-2}T^{-2}]$$

The dimensional formula for $A{\cdot}d$ is

$$[A{\cdot}d] = [MLT^{-2}][M^{-1}L^{-2}T^{-2}] = [L^{-1}]$$

23. (b) : Permissible error = ± least count = ± 0.1 cm

24. (d) : The error in g is minimum when the number of oscillations *i.e.* n is more.

$$\frac{\Delta g}{g} = \frac{\Delta l}{l}+\left|-2\frac{\Delta T}{T}\right| = \frac{\Delta l}{l}+\frac{2\Delta T}{T},$$

$\dfrac{\Delta l}{l}+\dfrac{2\Delta T}{T}$ should be minimum. Hence (d) represents the best measured data.

25. (a) : $[\text{Energy density}]=\left[\dfrac{\text{Work done}}{\text{Volume}}\right]$

$$=\frac{[MLT^{-2}L]}{[L^3]}=[ML^{-1}T^{-2}]$$

$$[\text{Young's modulus}]= [Y]=\left[\frac{\text{Force}}{\text{Area}}\right]\times\frac{[l]}{[\Delta l]}$$

$$=\frac{[MLT^{-2}]}{[L^2]}\frac{[L]}{[L]}=[ML^{-1}T^{-2}]$$

The dimensions of 1 and 4 are the same.

26. (a) : The dimensional formula of energy is $[ML^2T^{-2}]$.

$$n_2=1\left[\frac{1\text{ kg}}{10\text{ kg}}\right]^1\left[\frac{1\text{ m}}{1\text{ km}}\right]^2\left[\frac{1\text{ s}}{1\text{ min}}\right]^{-2}$$

$$=\frac{1}{10}\times\frac{1}{10^6}\times\frac{1}{(60)^{-2}}=\frac{3600}{10^7}=3.6\times10^{-4}$$

27. (b) : Planck's constant, $h=\dfrac{E}{\upsilon}$

$$=\frac{[ML^2T^{-2}]}{[T^{-1}]}=[ML^2T^{-1}]$$

Moment of inertia, $I = [ML^2]$

$$\frac{h}{I}=\frac{[ML^2T^{-1}]}{[ML^2]}=[T^{-1}]=\text{frequency}$$

28. (c) : According to the rules of significant figures,

1.64×10^{20} kg has three significant figures.

0.006 m^2 has one significant figure.

7.2180 J has five significant figures.

5.045 J has four significant figures.

29. (c) : The term $\dfrac{1}{2}\varepsilon_0E^2$ represents the energy per unit volume.

So, the dimensional formula is $\dfrac{[ML^2T^{-2}]}{[L^3]}$

or $\quad [ML^{-1}T^{-2}]$

30. (d) : Resistance $=\dfrac{\text{voltage}}{\text{current}}$ and

Capacitance $=\dfrac{\text{charge}}{\text{voltage}}$

$\therefore \quad$ Resistance × capacitance

$$=\frac{\text{voltage}}{\text{current}}\times\frac{\text{charge}}{\text{voltage}}=\frac{\text{charge}}{\text{current}}$$

$$=\frac{\text{current}\times\text{time period}}{\text{current}}=\text{time period.}$$

NCERT Xtract

New

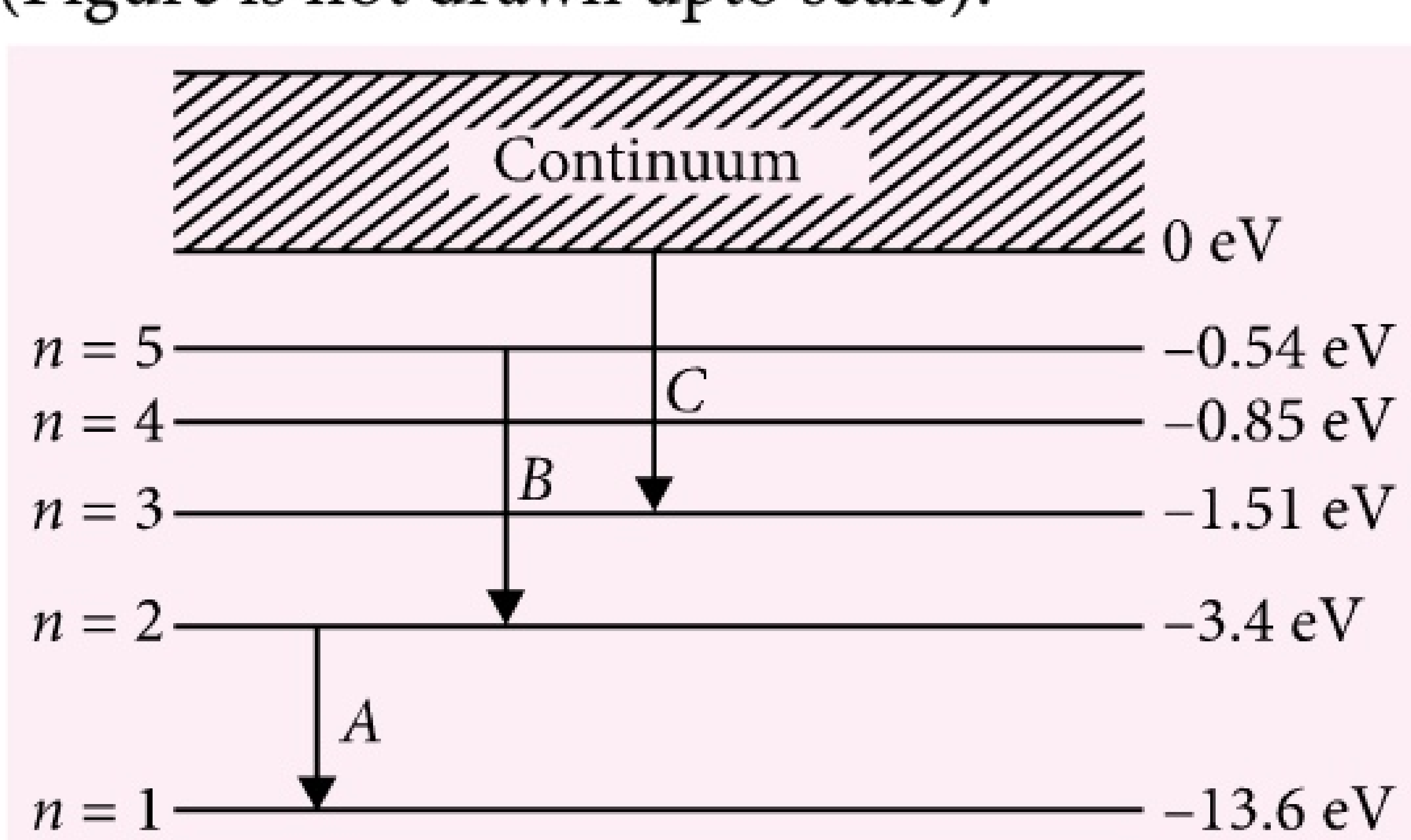
Questions for Medical/ Engineering Entrance Exams

Atoms

1. In Rutherford scattering experiment, the correct angle of scattering of alpha particles for impact parameter equal to zero is
(a) 0° (b) 90° (c) 180° (d) 270°

2. The total energy of an electron in the second excited state of the hydrogen atom is −1.51 eV. The kinetic and potential energies of the electron in this state are respectively
(a) 1.51 eV, −3.02 eV (b) −1.51 eV, 3.02 eV
(c) 3.02 eV, −1.51 eV (d) −3.02 eV, 1.51 eV

3. Figure shows the energy levels of the hydrogen atom along with some transitions marked as *A*, *B* and *C*. The transitions *A*, *B* and *C* respectively represent
(Figure is not drawn upto scale).



(a) the first line of Lyman series, second line of Balmer series and series limit of Paschen series.
(b) the second line of Lyman series, the second line of Balmer series and second line of Paschen series.
(c) the first line of Lyman series, the third line of Balmer series and series limit of Paschen series.
(d) the first line of Lyman series, second line of Balmer series and third line of Paschen series.

4. The angular momentum of an electron in any orbit of hydrogen atom is proportional to (r = radius of the orbit)
(a) $\frac{1}{\sqrt{r}}$ (b) $\frac{1}{r}$ (c) r^2 (d) $\sqrt{r}$

5. The de Broglie wavelength of an electron in 2nd orbit is (a_0 = Bohr radius)
(a) $2\pi a_0$ (b) $4\pi a_0$ (c) $8\pi a_0$ (d) $16\pi a_0$

6. What is the minimum energy that must be given to a H atom in ground state so that it can emit an H_γ line in Balmer series ?
(a) 13.06 eV (b) 12.75 eV
(c) 10.2 eV (d) 2.86 eV

7. The radius of Li^{++} ion in its ground state in terms of Bohr radius a_0 is
(a) a_0 (b) $\frac{a_0}{3}$ (c) $2a_0$ (d) $3a_0$

8. For which of the following atoms/ions Bohr model is not valid?
(a) H (b) He (c) Li^{++} (d) He^+

9. Two hydrogen atoms in the ground state collide inelastically. The maximum amount by which their combined kinetic energy is reduced is

(a) 10.2 eV (b) 20.4 eV
(c) 13.6 eV (d) 27.2 eV

10. The ratio of the speed of an electron in the ground state of hydrogen atom to the speed of light in vacuum is

(a) $\frac{1}{137}$ (b) $\frac{2}{137}$ (c) $\frac{1}{237}$ (d) $\frac{2}{237}$

11. The colour of first line of Balmer series is

(a) blue (b) green (c) violet (d) red

12. Bohr's theory of hydrogen atom is based on the

(a) quantisation of energy
(b) quantisation of charge
(c) quantisation of angular momentum
(d) quantisation of speed

13. An α-particle of energy 5 MeV is scattered through 180° by a gold nucleus. The distance of the closest approach is of the order of (For gold, $Z = 79$)

(a) 10^{-10} cm (b) 10^{-12} cm
(c) 10^{-14} cm (d) 10^{-16} cm

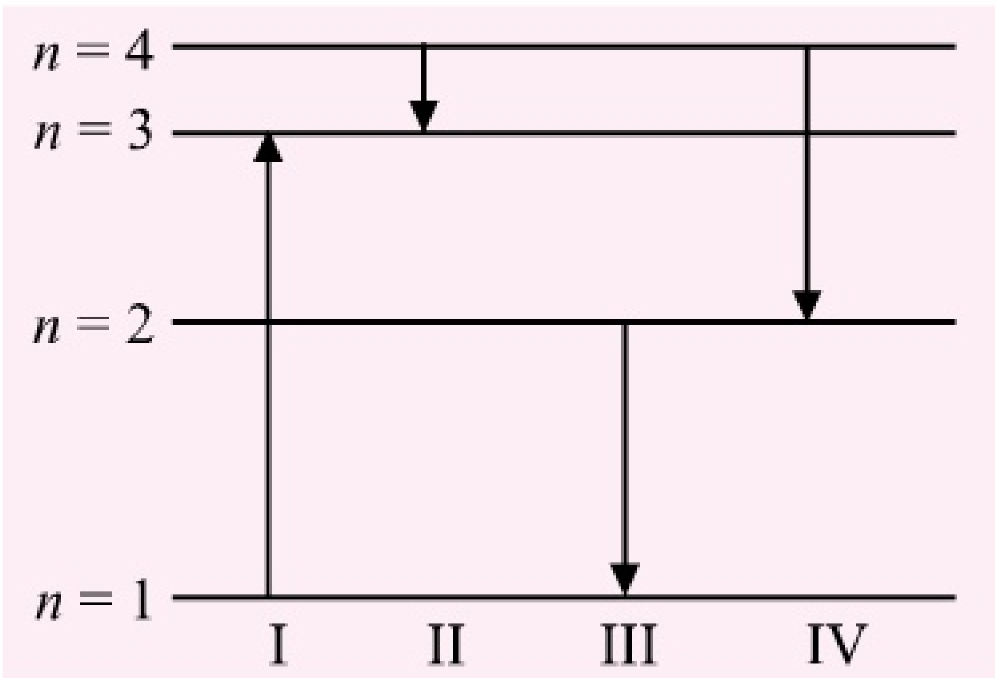
14. υ_1 is the frequency of the series limit of Lyman series, υ_2 is the frequency of the first line of Lyman series and υ_3 is the frequency of the series limit of the Balmer series. Then

(a) $\upsilon_1 - \upsilon_2 = \upsilon_3$ (b) $\upsilon_1 = \upsilon_2 - \upsilon_3$
(c) $\frac{1}{\upsilon_2} = \frac{1}{\upsilon_1} + \frac{1}{\upsilon_3}$ (d) $\frac{1}{\upsilon_1} = \frac{1}{\upsilon_2} + \frac{1}{\upsilon_3}$

15. The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because

(a) of the electrons not being subject to a central force.
(b) of the electrons colliding with each other.
(c) of screening effects.
(d) the force between the nucleus and an electron will no longer be given by Coulomb's law.

16. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?



(a) I (b) II (c) III (d) IV

17. In the Bohr model of the hydrogen atom, the lowest orbit corresponds to

(a) zero energy (b) minimum energy
(c) maximum energy (d) infinite energy

18. According to Bohr model of hydrogen atom, the electric current due to motion of electron in n^{th} orbit is proportional to

(a) $\frac{1}{n^3}$ (b) $\frac{1}{n^5}$ (c) n^3 (d) n^5

19. In the Bohr model of hydrogen atom, the ratio of the potential energy to the total energy of an electron in n^{th} state is

(a) –1 (b) 1 (c) –2 (d) 2

20. A 10 kg satellite circles earth once every 2 h in an orbit having a radius of 8000 km. Assuming that Bohr's angular momentum postulate applies to the satellite, then the quantum number of the orbit of the satellite is (Take $h = 6.6 \times 10^{-34}$ J s)

(a) 5.3×10^{47} (b) 5.3×10^{45}
(c) 5.3×10^{43} (d) 5.3×10^{41}

21. The H_β line of hydrogen

(a) has a wavelength 656.3 nm.
(b) has a wavelength 434.1 nm.
(c) has a wavelength smaller than that of H_γ line.
(d) is emitted in the transition from the third excited state to the first excited state.

22. Which of the following statements is incorrect regarding Rutherford's model of an atom?

(a) Most of the mass of the atom is concentrated at the nucleus.

(b) The size of the nucleus is very large as compared to the size of the atom.
(c) The electrons revolve around the nucleus in well-defined orbits.
(d) None of these

23. Which of the following parameters is the same for all hydrogen-like atoms and ions in their ground states?
(a) Radius of the orbit
(b) Speed of the electron
(c) Energy of the atom
(d) Orbital angular momentum of the electron

24. What is the wavelength of light for the least energetic photon emitted in the Lyman series of the hydrogen atom spectrum?
(Take $hc = 1240$ eV nm)
(a) 82 nm (b) 102 nm
(c) 122 nm (d) 150 nm

25. The energy of an excited hydrogen atom is -3.4 eV. The angular momentum of the electron is
(Take $h = 6.63 \times 10^{-34}$ J s)
(a) 1.05×10^{-34} J s (b) 2.11×10^{-34} J s
(c) 3.16×10^{-34} J s (d) 4.22×10^{-34} J s

26. The radius of the orbit of an electron in a hydrogen-like atom (atomic number Z) is $4.5a_0$ where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. The value of Z is
(a) 2 (b) 3 (c) 4 (d) 5

27. Which one of the following series of hydrogen spectrum is in ultraviolet region?
(a) Lyman (b) Balmer
(c) Paschen (d) Brackett

28. In Rutherford's scattering experiment, the number of α-particles scattered through an angle of 60° is 100 per minute. Then the number of particles scattered through an angle of 90° per minute by the same nucleus is
(a) 20 (b) 25 (c) 50 (d) 100

29. In the Bohr model of a hydrogen atom, the centripetal force is furnished by the Coulomb attraction between the proton and the electron. If a_0 is the radius of the ground state orbit, m is the mass and e is the charge on the electron and ε_0 is the permittivity of vacuum, the speed of the electron is

(a) $\frac{\sqrt{\varepsilon_0 a_0 m}}{e}$ (b) $\frac{e}{\sqrt{\varepsilon_0 a_0 m}}$

(c) $\frac{e}{\sqrt{4\pi\varepsilon_0 a_0 m}}$ (d) $\sqrt{\frac{4\pi\varepsilon_0 a_0 m}{e}}$

30. In Bohr theory of hydrogen atom, let r, v and E be the radius of orbit, speed of electron and the total energy of the electron respectively. Which of the following quantities is proportional to the quantum number n?
(a) vr (b) rE (c) $\frac{r}{E}$ (d) $\frac{r}{v}$

HIGHER ORDER THINKING SKILLS QUESTIONS (HOTS)

31. Electrons are bombarded to excite hydrogen atoms and six spectral lines are observed. If E_g is the ground state energy of hydrogen, the minimum energy the bombarding electrons should possess is

(a) $\frac{8E_g}{9}$ (b) $\frac{15E_g}{16}$ (c) $\frac{35E_g}{36}$ (d) $\frac{48E_g}{49}$

32. A hydrogen sample is prepared in a particular excited state A. Photons of energy 2.55 eV get absorbed into the sample to take some of the electrons to a further excited state B. The quantum numbers corresponding to the states A and B respectively are
(a) 2 and 4 (b) 3 and 5
(c) 3 and 6 (d) 4 and 7

33. The energy of an electron in the n^{th} orbit of positronium is

(a) $-\frac{13.6}{n^2}$ eV (b) $-\frac{27.2}{n^2}$ eV

(c) $-\frac{27.2}{2n^2}$ eV (d) $-\frac{13.6}{2n^2}$ eV

34. The radius of the hydrogen atom in its ground state is a_0. The radius of a muonic hydrogen atom in which the electron is replaced by an identically charged muon with mass 207 times that of an electron, a_μ is equal to

(a) $207a_0$ (b) $\frac{a_0}{207}$

(c) $\frac{a_0}{\sqrt{207}}$ (d) $a_0\sqrt{207}$

35. Hydrogen atom in its ground state is excited by means of a monochromatic radiation of wavelength 970.6 Å . How many different wavelengths are possible in the resulting emission spectrum? (Take $h = 6.6 \times 10^{-34}$ J s)

(a) 2 (b) 4 (c) 6 (d) 8

SOLUTIONS

1. (c): For angle of scattering, $\theta = 180°$, the impact parameter is zero.

2. (a) : In the second excited state,

Total energy, $E = -1.51$ eV

Kinetic energy, $K = -E = -(-1.51 \text{ eV}) = 1.51$ eV

Potential energy, $U = 2E = 2(-1.51 \text{ eV})$

$= -3.02$ eV

3. (c): A represents the first line of Lyman series, B represents the third line of Balmer series and C represents series limit of Paschen series.

4. (d) : According to Bohr's quantisation condition

Angular momentum, $L = \frac{nh}{2\pi}$

As $r \propto n^2$,

Hence, $L \propto \sqrt{r}$.

5. (b) : Radius of n^{th} orbit is

$r_n = n^2 a_0$ where a_0 is, the Bohr radius.

If λ is the de Broglie wavelength of an electron while revolving in n^{th} orbit of radius r_n, then

$2\pi r_n = n\lambda$

$$\lambda = \frac{2\pi r_n}{n}$$

For 2^{nd} orbit, $n = 2$

$$\therefore \quad \lambda = \frac{2\pi r_2}{2} = \frac{2\pi(2)^2 a_0}{2} = 4\pi a_0$$

6. (a) : H_γ line in the Balmer series corresponds to transition from $n = 5$ to $n = 2$.

Therefore, electron in the ground state ($n = 1$) must be raised first to the state ($n = 5$).

The minimum energy required for this purpose is

$E = E_5 - E_1$

$$= -\frac{13.6}{5^2}\text{eV} - \left(-\frac{13.6}{1^2}\text{eV}\right)\left(\text{As } E_n = -\frac{13.6}{n^2}\text{eV}\right)$$

$= -0.54$ eV $+ 13.6$ eV $= 13.06$ eV

7. (b) : Radius of n^{th} orbit of hydrogen-like atom is

$$r_n = \frac{n^2}{Z} a_0$$

For Li^{++} ion, $Z = 3$, $n = 1$ for ground state

$\therefore$ Radius of Li^{++} ion in its ground state is

$$r_1 = \frac{(1)^2}{3} a_0 = \frac{a_0}{3}$$

8. (b) : Bohr model is valid only for one-electron atoms/ions.

Bohr model is not valid in He atom because it has two electrons.

9. (a) : Initial kinetic energy of each of two hydrogen atoms in ground state = 13.6 eV

$\therefore$ Kinetic energy of both H atoms before collision = 2 × 13.6 eV= 27.2 eV

As the collision is inelastic, linear momentum is conserved but some kinetic energy is lost.

If one H atom goes to first excited state and other remains in ground state, then their combined kinetic energy after collision

$$= \frac{13.6}{2^2}\text{ eV} + 13.6\text{ eV} = 17\text{ eV}$$

$\therefore$ Reduction in their combined kinetic energy = 27.2 eV – 17 eV = 10.2 eV

10. (a) : Speed of an electron in the ground state of hydrogen atom is

$$v = \frac{1}{4\pi\varepsilon_0}\frac{2\pi e^2}{h} = c\left(\frac{1}{4\pi\varepsilon_0}\frac{2\pi e^2}{ch}\right) = \alpha c = \frac{c}{137}$$

where c = speed of light in vacuum

$$\alpha = \frac{1}{4\pi\varepsilon_0}\frac{2\pi e^2}{ch} = \frac{1}{137} = \text{Fine structure constant}$$

$$\therefore \quad \frac{v}{c} = \frac{1}{137}$$

11. (d) : The colour of first line of Balmer series is red.

12. (c): Bohr's theory of hydrogen atom is based on quantisation of angular momentum.

13. (b) : At the distance of closest approach d,
Kinetic energy of an alpha particle
= Potential energy of alpha particle and the gold nucleus.

$$K = \frac{1}{4\pi\varepsilon_0}\frac{(2e)(Ze)}{d}$$

$$d = \frac{1}{4\pi\varepsilon_0}\frac{2Ze^2}{K}$$

Here,

$K = 5$ MeV $= 5 \times 10^6$ eV
$= 5 \times 10^6 \times 1.6 \times 10^{-19}$ J
$= 5 \times 1.6 \times 10^{-13}$ J
$Z = 79$

$$\therefore \quad d = \frac{9\times10^9 \times 2\times 79\times(1.6\times10^{-19})^2}{5\times1.6\times10^{-13}}$$

$= 4.55 \times 10^{-14}$ m ~ 10^{-12} cm

14. (a) : For Lyman series

$$\upsilon = Rc\left[\frac{1}{1^2} - \frac{1}{n^2}\right] \text{ where } n = 2, 3, 4, ...$$

For the series limit of Lyman series, $n = \infty$

$$\therefore \quad \upsilon_1 = Rc\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = Rc \qquad \text{...(i)}$$

For the first line of Lyman series, $n = 2$

$$\therefore \quad \upsilon_2 = Rc\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4}Rc \qquad \text{...(ii)}$$

For Balmer series

$$\upsilon = Rc\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \text{ where } n = 3, 4, 5....$$

For the series limit of Balmer series, $n = \infty$

$$\therefore \quad \upsilon_3 = Rc\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] = \frac{Rc}{4} \qquad \text{...(iii)}$$

From equations (i), (ii) and (iii), we get
$\upsilon_1 = \upsilon_2 + \upsilon_3$ or $\upsilon_1 - \upsilon_2 = \upsilon_3$

15. (a) : Each electron interacts not only with the positively charged nucleus (which provides the central force) but also with all other electrons *i.e.* we have to take into account electron-electron interaction. Thus, the electrons are not being subjected to a central force.

16. (c): Ist transition is showing absorption of a photon. From rest of three transitions, III is having maximum energy from level $n = 2$ to $n = 1$

$$\Delta E \propto \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

17. (b) : In hydrogen atom, the lowest orbit corresponds to minimum energy.

18. (a) : Electric current due to motion of electron in n^{th} orbit is
$I_n = e\upsilon_n$
For hydrogen atom,
Frequency of electron in n^{th} orbit is

$$\upsilon_n = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4\pi^2 e^4 m}{n^3 h^3} \qquad \text{...(i)}$$

$$\therefore \quad I_n = \left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4\pi^2 e^5 m}{n^3 h^3} \qquad \text{(Using(i))}$$

Thus, $I_n \propto \frac{1}{n^3}$

19. (d) : For hydrogen atom,
The kinetic energy of the electron in n^{th} state is

$$K = \frac{me^4}{8\varepsilon_0^2 h^2 n^2} = \frac{13.6}{n^2} \text{ eV}$$

where $\frac{me^4}{8\varepsilon_0^2 h^2} = 13.6$ eV

The potential energy of the electron in n^{th} state is

$$U = -\frac{2me^4}{8\varepsilon_0^2 h^2 n^2} = -\frac{27.2}{n^2} \text{ eV}$$

Total energy of the electron in n^{th} state is

$$E = K + U = \frac{me^4}{8\varepsilon_0^2 h^2 n^2} - \frac{2me^4}{8\varepsilon_0^2 h^2 n^2}$$

$$= -\frac{me^4}{8\varepsilon_0^2 h^2 n^2} = -\frac{13.6}{n^2} \text{ eV}$$

$$\therefore \quad \frac{U}{E} = 2$$

20. (b) : Here, $m = 10$ kg,
$r = 8000$ km $= 8000 \times 10^3$ m $= 8 \times 10^6$ m,
$T = 2$ h $= 7200$ s
According to Bohr's second postulate

$$L = mvr = \frac{nh}{2\pi}$$

As $v = \dfrac{2\pi r}{T}$

$$\therefore \quad m\left(\frac{2\pi r}{T}\right)r = \frac{nh}{2\pi}$$

$$n = \frac{(2\pi r)^2 m}{Th}$$

$$= \frac{(2 \times 3.14 \times 8 \times 10^6)^2 \times 10}{7200 \times 6.6 \times 10^{-34}} = 5.3 \times 10^{45}$$

21. (d) : The wavelength of H_β line is 486.1 nm.
The wavelength of H_β line is greater than that of H_γ line ($\lambda_{H_\gamma} = 434.1$ nm).
H_β line corresponds to transition from the third excited state ($n = 4$) to the first excited state ($n = 2$).

22. (b) : The size of the nucleus ($\approx 10^{-15}$ m) is very small as compared to the size of the atom ($\approx 10^{-10}$ m).

23. (d)

24. (c) : For any series, the transition that produces the least energetic photon is the transition between the home-base level that defines the series and the level immediately above it.
For the Lyman series, the home-base level is at $n = 1$. Thus, the transition that produces the least energetic photon is the transition from the $n = 2$ level to the $n = 1$ level.

As $E_n = -\dfrac{13.6}{n^2}$ eV

$$\therefore \quad E_1 = -\frac{13.6}{1^2} \text{ eV} = -13.6 \text{ eV}$$

and $E_2 = -\dfrac{13.6}{2^2}$ eV $= -3.4$ eV

The corresponding energy difference is

$$\Delta E = E_2 - E_1$$
$$= -3.4 \text{ eV} - (-13.6 \text{ eV}) = 10.2 \text{ eV}$$

Wavelength of emitted photon is

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV nm}}{10.2 \text{ eV}} = 122 \text{ nm}$$

25. (b) : The energy of n^{th} state of hydrogen atom is

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

$$\therefore \quad n^2 = \frac{-13.6 \text{ eV}}{E_n} = \frac{-13.6 \text{ eV}}{-3.4 \text{ eV}} = 4$$

($\because E_n = -3.4$ eV)

$n = 2$

According to Bohr's quantisation condition

Angular momentum, $L = \dfrac{nh}{2\pi} = \dfrac{2h}{2\pi} = \dfrac{h}{\pi}$

$$= \frac{6.63 \times 10^{-34} \text{ J s}}{3.14}$$

$= 2.11 \times 10^{-34}$ J s

26. (a) : According to Bohr's quantisation condition

$$L = \frac{nh}{2\pi}$$

Given : $L = \dfrac{3h}{2\pi}$

$$\therefore \quad \frac{3h}{2\pi} = \frac{nh}{2\pi} \text{ or } n = 3$$

Radius of n^{th} orbit for hydrogen-like atom is

$r_n = \dfrac{n^2}{Z} a_0$ where a_0 is the Bohr radius

Here, $r_n = 4.5a_0$

$$\therefore \quad 4.5a_0 = \frac{(3)^2}{Z} a_0 \text{ or } Z = \frac{9}{4.5} = 2$$

27. (a) : The Lyman series is in the ultraviolet region , Balmer series is in the visible region, and the Paschen and Brackett are in the infrared region.

28. (b) : According to Rutherford's scattering formula

$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)}$$

Thus, $\frac{N(90°)}{N(60°)} = \frac{\sin^4(60°/2)}{\sin^4(90°/2)} = \frac{\sin^4 30°}{\sin^4 45°}$

$$= \frac{(\sin 30°)^4}{(\sin 45°)^4} = \frac{(1/2)^4}{(1/\sqrt{2})^4} = \frac{1}{4}$$

$N(90°) = \frac{1}{4} N(60°) = \frac{1}{4} \times 100 = 25$

29. (c): $\frac{mv^2}{a_0} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{a_0^2}$

$\therefore \quad v = \frac{e}{\sqrt{(4\pi\varepsilon_0 a_0 m)}}$

30. (a) : According to Bohr theory of hydrogen atom

$r \propto n^2,\ v \propto \frac{1}{n},\ E \propto \frac{1}{n^2} \quad \therefore \quad vr \propto n$

31. (b) : To obtain 6 spectral lines, as electron must be excited to fourth orbit with energy $\frac{E_g}{16}$, so that the energy difference is $\frac{15E_g}{16}\left(= E_g - \frac{E_g}{16}\right)$

Hence, the minimum energy the bombarding electrons should possess is $\frac{15E_g}{16}$.

32. (a) : The allowed energies of hydrogen atom are

$E_1 = -13.6$ eV
$E_2 = -3.4$ eV
$E_3 = -1.51$ eV
$E_4 = -0.85$ eV
$E_5 = -0.54$ eV and so on.

A energy difference of 2.55 eV can only be absorbed in transition $n = 2$ to $n = 4$. So, the state A has quantum number 2 and the state B has quantum number 4.

33. (d)

34. (b) : $a_0 = \frac{h^2\varepsilon_0}{\pi me^2}$ … (i)

$a_\mu = \frac{h^2\varepsilon_0}{\pi(207m)e^2}$ … (ii)

Dividing (ii) by (i), we get

$\frac{a_\mu}{a_0} = \frac{1}{207}$ or $a_\mu = \frac{a_0}{207}$

35. (c): Here, $\lambda = 970.6$ Å $= 970.6 \times 10^{-10}$ m

Energy of the monochromatic radiation,

$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{970.6\times10^{-10}\times1.6\times10^{-19}} \text{ eV}$$
$$= 12.75 \text{ eV}$$

Energy of the hydrogen atom is

$E_n = -\frac{13.6}{n^2}$ eV

For ground state, $n = 1$

$\therefore \quad E_1 = -13.6$ eV

Energy of the hydrogen atom after excitation of the state (n say) *i.e.*,

$E_n = E_1 + E$
$= -13.6 \text{ eV} + 12.75 \text{ eV} = -0.85 \text{ eV}$

As $E_n = -\frac{13.6}{n^2}$ eV

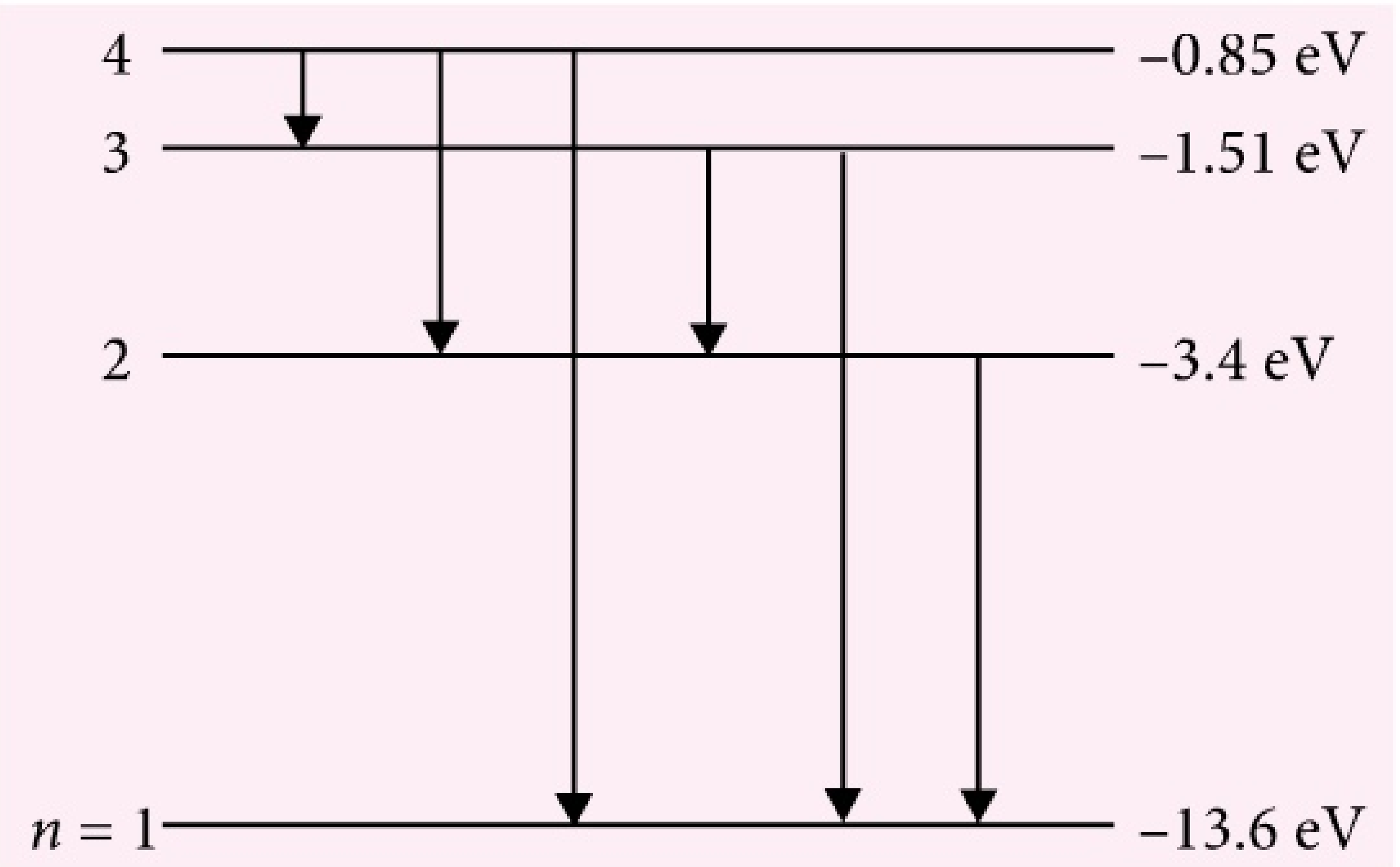
$\therefore \quad -0.85 \text{ eV} = -\frac{13.6}{n^2}$ eV

$n^2 = \frac{13.6}{0.85} = 16$
$n = 4$

Thus, the hydrogen atom is excited to $n = 4$ state.

$\therefore$ Total number of wavelengths in emission spectrum $= \frac{(n)(n-1)}{2} = \frac{(4)(3)}{2} = 6.$

The possible emission lines are as shown in the figure.



PHYSICS MUSING

SOLUTION SET-12

1. **(c):** Here $R = R_0 - \beta t$

Radial component of velocity is

$$v_r = \frac{dR}{dt} = \frac{d}{dt}(R_0 - \beta t) = -\beta$$

$$v_t = \omega R = \omega(R_0 - \beta t) = R_0 - \beta t$$

$$(\because \omega = 1 \text{ rad s}^{-1})$$

$$\therefore \quad v = \sqrt{v_r^2 + v_t^2} = \sqrt{\beta^2 + (R_0 - \beta t)^2}$$

2. **(a)** **:** Work done by atmosphere, $W = P_{atm}\Delta V$

or $W = P_{atm}\dfrac{V}{2}$ $\quad (\because \Delta V = V - \dfrac{V}{2})$ $\quad$ ...(i)

Initially gas in the container is in thermodynamic equilibrium with its surroundings.

$\therefore$ Pressure inside cylinder = P_{atm}

$$PV = nRT$$

$$\Rightarrow P_{atm}V = nRT \quad \text{or} \quad V = \frac{nRT}{P_{atm}}$$

Putting in equation (i), we get

$$W = \frac{nRT}{2}$$

3. **(d)** **:** As $I = \dfrac{MR^2}{2} - \left[\dfrac{mr^2}{2} + ma^2\right]$

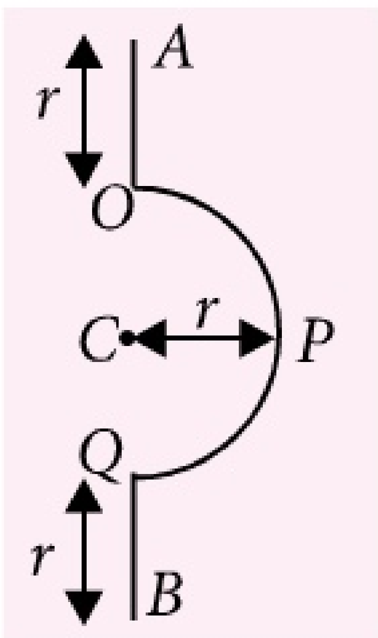
Here, $R = 4$ m, $r = 1$ m and $a = 3$ m

$M = 2$ kg and $m = \dfrac{1}{8}$ kg

$$\therefore \quad I = \frac{2\times 16}{2} - \frac{1}{8}\left(\frac{1}{2} + 9\right)$$

$$= 16 - \frac{19}{16} = \frac{237}{16} \text{ kg m}^2$$

4. **(a)** **:** Let λ be the mass per unit length of wire.
Moment of inertia of OA or QB,



$$I_1 = \frac{\lambda r \times r^2}{12} + \lambda r \times \left(\frac{3r}{2}\right)^2 = \frac{7\lambda r^3}{3}$$

Moment of inertia of OPQ, $I_2 = \lambda\pi r \times r^2$

$$I = 2I_1 + I_2 = \frac{14\lambda r^3}{3} + \lambda\pi r^3$$

Here $\lambda = \dfrac{M}{\pi r + 2r}$

$$\therefore \quad I = \left(\frac{14 + 3\pi}{3}\right) r^3 \times \frac{M}{(\pi + 2)r}$$

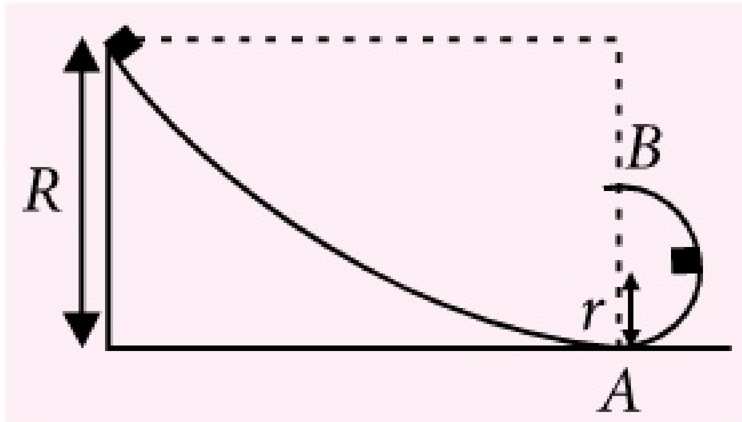
$$= Mr^2\left(\frac{14 + 3\pi}{3\pi + 6}\right)$$

5. **(a)** **:** From conservation of linear momentum,
$mv - mv + 0 = 3mv_{cm}$
$\therefore$ $v_{cm} = 0$
From conservation of angular momentum,

$$2mv \times \frac{l}{2} = \left(\frac{ml^2}{12} + \frac{2ml^2}{4}\right)\omega$$

$$\therefore \quad \omega = \frac{12v}{7l}$$

6. (c):



At the highest point of inner loop,

$$mg + N = \frac{mv^2}{r} \Rightarrow N = \frac{mv^2}{r} - mg$$

The block may not loose contact if $N \geq 0$

$$\frac{mv^2}{r} - mg \geq 0$$

or $\quad mv^2 \geq mgr \quad$ ... (i)

From conservation of energy

$$\frac{1}{2}mv^2 + 2mgr = mgR$$

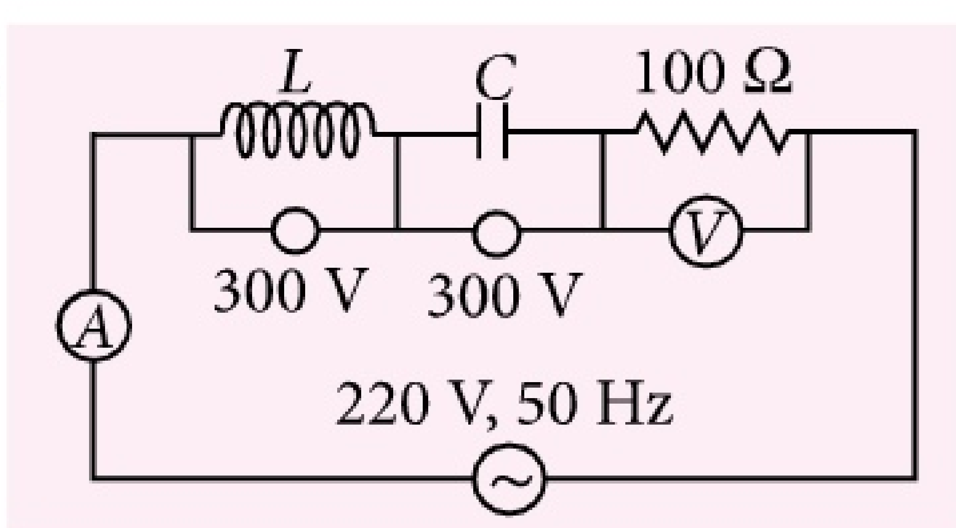
$\Rightarrow \quad mv^2 = 2mgR - 4mgr$

or $\quad mgr \leq 2\,mgR - 4mgr \quad$ [Using eqn. (i)]

or $\quad 5r \leq 2R$

or $\quad \dfrac{R}{r} \geq \dfrac{5}{2} \quad$ or $\quad \left(\dfrac{R}{r}\right)_{\min} = \dfrac{5}{2}$

7. (b) :



As $\; V_L = V_C = 300$ V,

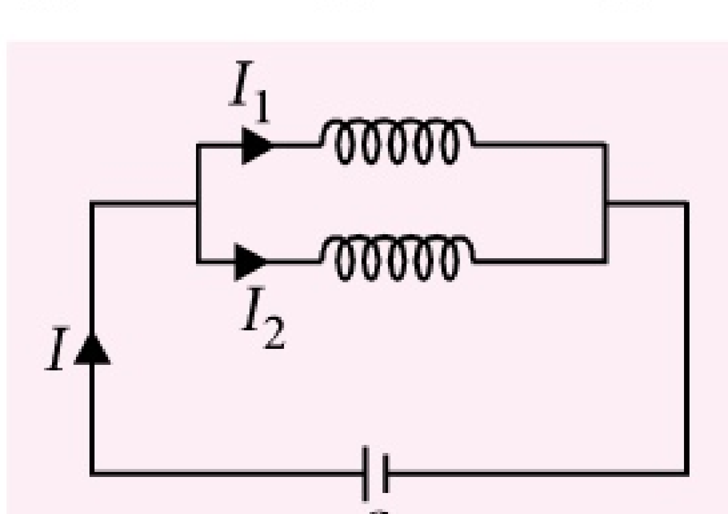
and $\; V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$\therefore \quad V_R = V = 220$ V

Also $\; I = \dfrac{V}{R} = \dfrac{220}{100} = 2.2$ A

8. (a) : Let L_{eq} be the combined inductance.

$$\therefore \quad \varepsilon = L_{eq}\frac{dI}{dt} = L_1\frac{dI_1}{dt} = L_2\frac{dI_2}{dt}$$



Also, $I = I_1 + I_2$

or $\quad \dfrac{dI}{dt} = \dfrac{dI_1}{dt} + \dfrac{dI_2}{dt} \Rightarrow \dfrac{\varepsilon}{L_{eq}} = \dfrac{\varepsilon}{L_1} + \dfrac{\varepsilon}{L_2}$

$$\therefore \quad \frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

$$\Rightarrow \quad L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$

9. (a) : The equivalent capacitance in each case are as follows :

(a) $\; \dfrac{1}{C} = \dfrac{1}{5 \times 2} + \dfrac{2}{2} = \dfrac{11}{10} \quad$ or $\quad C = \dfrac{10}{11}\,\mu$F

(b) $\; \dfrac{1}{C} = \dfrac{1}{4 \times 2} + \dfrac{3}{2} = \dfrac{13}{8} \quad$ or $\quad C = \dfrac{8}{13}\,\mu$F

(c) $\; \dfrac{1}{C} = \dfrac{1}{3 \times 2} + \dfrac{4}{2} = \dfrac{13}{6} \quad$ or $\quad C = \dfrac{6}{13}\,\mu$F

(d) $\; \dfrac{1}{C} = \dfrac{1}{2 \times 2} + \dfrac{5}{2} = \dfrac{11}{4} \quad$ or $\quad C = \dfrac{4}{11}\,\mu$F

10. (a) : Current through the circuit,

$$I = \frac{(n-1)\varepsilon - \varepsilon}{nr} = \frac{(n-2)\varepsilon}{nr}$$

Potential drop across each cell, except A, is

$$\varepsilon - Ir = \varepsilon - \frac{(n-2)\varepsilon}{nr} \times r$$

$$= \varepsilon - \varepsilon + \frac{2\varepsilon}{n} = \frac{2\varepsilon}{n}$$